Management Science

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MANAGEMENT SCIENCE
Articles in Advance, pp. 1–21
ISSN 0025-1909 (print) | ISSN 1526-5501 (online)

http://dx.doi.org/10.1287/mnsc.2015.2378

Dynamic Pricing in the Presence of Social Learning and Strategic Consumers

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When a product of uncertain quality is first introduced, consumers may choose to strategically delay their purchasing decisions in anticipation of the product reviews of their peers. This paper investigates how the presence of social learning affects the strategic interaction between a dynamic-pricing monopolist and a forward-looking consumer population, within a simple two-period model. Our analysis yields three main insights. First, we find that the presence of social learning has significant structural implications for optimal pricing policies: In the absence of social learning, decreasing price plans are always preferred by the firm; by contrast, in the presence of social learning we find that (i) if the firm commits to a price path ex ante (preannounced pricing), an increasing price plan is typically announced, whereas (ii) if the firm adjusts price dynamically (responsive pricing), prices are initially low and may either rise or decline over time. Second, we establish that under both preannounced and responsive pricing, even though the social learning process exacerbates strategic consumer behavior (i.e., increases strategic purchasing delays), its presence results in an increase in expected firm profit. Third, we illustrate that, contrary to results reported in existing literature on strategic consumer behavior, in settings where social learning is significantly influential, preannounced pricing policies are generally not beneficial for the firm.

Keywords: Bayesian social learning; strategic consumer behavior; dynamic pricing; applied game theory
History: Received January 4, 2014; accepted July 19, 2015, by Yossi Aviv, operations management. Published online in *Articles in Advance*.

1. Introduction

The term "strategic consumer" is commonly used in the literature to describe a rational and forward-looking consumer who makes intertemporal purchasing decisions with the goal of maximizing her utility. In its simplest form, strategic behavior may manifest as bargain-seeking behavior, whereby even if the current price of a product is lower than the customer's willingness to pay, she may delay her purchase in anticipation of a future markdown.[1] The importance of forward-looking consumer behavior in shaping firms' pricing decisions has been widely recognized by practitioners and academics alike: to defend against its negative effects, firms are investing heavily in price-optimization algorithms (e.g., Schlosser 2004), while the literature has produced a number of managerial insights regarding how firms should adjust their approach to dynamic pricing (e.g., Aviv and Pazgal 2008, Besbes and Lobel 2015, Cachon and Feldman 2015, Mersereau and Zhang 2012, Su 2007). Apart from pricing, the effects of strategic consumer behavior also extend to a range of other operational decisions; examples include decisions pertaining to stocking quantities (Liu and van Ryzin 2008), inventory display formats (Yin et al. 2009), the implementation of quick-response and fast-fashion practices (Cachon and Swinney 2009, 2011), and the timing of new product launches (Lobel et al. 2015), to name but a few. Although existing research examines strategic consumer behavior from a variety of perspectives, it generally does not account for cases in which the quality of a new product is ex ante uncertain and, more importantly, for the prominent role of social learning (SL) in resolving this uncertainty.

In reality, many new product introductions are accompanied by quality uncertainty, in particular owing to the ever-increasing complexity of product features. Examples of such products include high-tech consumer electronics (e.g., smartphones, tablets, computers), media items (e.g., movies, books), and digital products (e.g., computer software, smartphone apps). In the post-Internet era, online platforms hosting buyer-generated product reviews offer a cheap and straightforward way of reducing quality uncertainty. For the consumers, learning from reviews allows for better-informed purchasing decisions, which in turn reduces the likelihood

[1] See Li et al. (2014) for empirical evidence of strategic consumer behavior in the air-travel industry.

of ex post negative experiences. For the firm, the SL process can also be beneficial, for instance, by allowing for increased accuracy in forecasting future demand (e.g., Dellarocas et al. 2007). However, the ease with which the modern-day consumer can gain access to buyer reviews also gives rise to a new dimension of strategic consumer behavior: rather than experimenting with a new product themselves, consumers are enticed to delay their purchasing decisions in anticipation of the reviews of their peers (*The Economist* 2009). As a result, both the learning process (in terms of information generation) and the firm's performance (in terms of product adoption and profit) may be significantly hampered.

Despite the well-documented importance of managing strategic consumer behavior, our understanding of the effectiveness of alternative operational decisions in settings where SL is influential is extremely limited. In this paper, we take one step toward developing such an understanding by considering the fundamental problem of uncapacitated dynamic pricing. Our goal is to investigate how the presence of SL changes the strategic interaction between a monopolist firm and a population of consumers, with a particular emphasis on three research questions. First, how are the firm's pricing decisions altered to accommodate the SL process? We are interested in understanding and illustrating the main drivers underlying the optimal implementation of dynamic pricing, when the firm faces strategic consumers who interact socially through product reviews. Second, what is the impact of SL on the firm's profit? Existing research suggests that the presence of SL is beneficial for the firm (e.g., Ifrach et al. 2015); however, this work typically does not account for the potentially detrimental effects of strategic consumer behavior. Third, should a firm facing strategic consumers commit to a price path ex ante or adjust prices dynamically over time? The issue of price commitment is one that arises frequently in the strategic consumer literature, where preannounced price plans are generally found to be beneficial for the firm (e.g., Aviv and Pazgal 2008).

The model setting we consider is much in the spirit of the seminal paper by Besanko and Winston (1990). There is a monopolist firm selling a new product to a fixed population of strategic consumers, over two periods. Two alternative classes of dynamic-pricing policies may be employed: the firm may either (a) announce the full price path from the beginning of the selling horizon (*preannounced* pricing) or (b) announce only the first-period price and delay the second-period price announcement until the beginning of the second period (*responsive* pricing).[2] Consumers are heterogeneous in their preferences for the product and make adoption decisions to maximize their expected utility. Our addition to this simple model, and the focal point of our analysis, is the introduction of ex ante quality uncertainty (faced by both the firm and the consumers), which may be partially resolved in the second period by observing the reviews of first-period buyers (SL).

Because in the presence of SL the product's quality is partially learned in the second period, the interaction between the firm and the consumers is transformed from a game whose outcome can be predicted from the onset (in the absence of SL) to one whose outcome is of a probabilistic nature (i.e., a stochastic game). For the firm, the SL process generates demand uncertainty because first-period reviews generate an ex ante probabilistic shift in the second-period demand curve. For the consumers, SL offers an opportunity to better learn the value of the product, should they choose to delay their purchasing decision. Crucially, both the firm's pricing decisions and the consumers' adoption decisions are complicated by the fact that the generation of product information is endogenous to consumers' adoption decisions; for instance, if no sales occur in the first period, then no reviews are generated, and therefore nothing is learned by consumers who delay their purchasing decision.

Under either pricing regime, we show that conditional on the firm's first-period announcement, the equilibrium in the pricing-adoption game is unique. To distill the effects of SL on the game between the firm and the consumers, we compare the equilibrium outcomes of our model against those of a benchmark model in which the firm and the consumers remain forward-looking, but in which we "switch off" the SL process (i.e., by cutting off the firm's and the consumers' access to product reviews). This comparison yields three main sets of insights, which we summarize below.

First, we identify significant implications for the optimal implementation of dynamic pricing. When the firm employs preannounced pricing, in the absence of SL, it is always optimal to announce a decreasing price path (i.e., to employ "price skimming"). By contrast, in the presence of SL, the firm finds it optimal to announce an increasing price plan (unless consumers are nonstrategic). The intuition underlying this result is associated with the firm's desire to deter strategic purchasing delays (by making consumers "pay" for using review information) while at the same time extracting high rents in favorable SL scenarios (through

[2] Preannounced dynamic pricing is commonly employed in practice indirectly; for instance, firms may set a regular price and offer introductory price cuts (e.g., via promotional offers or coupons). Moreover, maintaining a constant price is also a special case of a preannounced price plan. On the other hand, responsive pricing (sometimes referred to in the literature as "contingent pricing") is commonly observed in online commerce; for example, Amazon.com is known to employ complex dynamic-pricing algorithms (Vanek Smith 2012).

the high second-period price). When the firm employs responsive pricing, the first-period price is decreased in the presence of SL, and the second-period price is ex ante random. The lower introductory price represents the firm's response to consumers' increased tendency to delay purchase in the presence of SL, whereas the ex ante uncertain nature of the second-period price reflects how the firm's second-period pricing decision is adapted to the content of the reviews generated by first-period buyers—both increasing and decreasing price plans occur with positive probability.

Second, we establish that the presence of SL is ex ante beneficial for the firm under both preannounced and responsive pricing, even when the consumer population is highly strategic.[3] This result is not immediately obvious, because the consumers' strategic behavior is exacerbated by the presence of SL: the opportunity to learn from product reviews gives rise to a "free-riding" effect, which increases the number of adoption delays over and above those observed in the absence of SL. Nevertheless, we find that the beneficial informational effect of SL more than compensates for this detrimental behavioral effect. Interestingly, this is true even under preannounced pricing, where the firm has no direct benefit from the learning process (since the full price path is decided before any reviews are generated). This result generalizes pre-existing findings that the presence of SL increases firm profit when consumers are nonstrategic (e.g., Ifrach et al. 2015).

Third, we provide insight into which class of policies is preferred by the firm when facing strategic consumers. A general finding of existing research is that responsive pricing, despite its inherent flexibility, can be suboptimal for the firm owing to the interplay between the product's price path and the purchasing decisions of the strategic consumers (e.g., Aviv and Pazgal 2008, Tang 2006). Our benchmark model concurs with the optimality of preannounced pricing policies. However, once SL is introduced into the model, our analysis and numerical experiments indicate that this finding is reversed: in the presence of SL, the firm prefers a responsive price plan (unless consumers are highly strategic *and* product reviews are not very informative). Furthermore, we observe that the presence of SL has the beneficial effect of aligning the firm's and the consumers' preferences for the type of policy that is chosen by the firm: in the absence of SL, the firm prefers a preannounced price plan, whereas consumers prefer a responsive price plan; in the presence of SL, responsive pricing is preferred by both. As a result, the presence of SL ensures that the class of policies employed by the firm is that which achieves higher total welfare.

2. Related Literature

The literature that considers strategic consumer behavior typically assumes that firms employ one of two classes of dynamic-pricing policies, either (i) preannounced or (ii) responsive pricing.[4] For early work focusing on the implications of each of the two classes of policies, we refer the reader to Stokey (1979) and Landsberger and Meilijson (1985) for preannounced pricing, and to Besanko and Winston (1990) for responsive pricing. Since then, both classes have been used extensively to study various operational decisions; for instance, Yin et al. (2009) and Whang (2015) use preannounced pricing to study the implications of alternative inventory display formats and demand learning, respectively, whereas Cachon and Swinney (2009) examine the firm's quantity and salvage-pricing decisions under responsive pricing. This paper is a first attempt toward understanding the relative effectiveness of preannounced and responsive pricing when the firm and the consumers face quality uncertainty that can be resolved through SL. As such, our model and analysis are much in the spirit of Landsberger and Meilijson (1985) and Besanko and Winston (1990), in that our focus is on highlighting the effects of SL within a simple model of the interactions between the firm and the consumer population. Our analysis demonstrates that the implications of SL are significant under both preannounced and responsive pricing.

A question of particular interest in our work is which class of policies (i.e., preannounced or responsive) is preferred by the firm. Responsive price plans generate value because they allow the firm to react optimally to updated information (e.g., demand forecasts, leftover inventory; see Elmaghraby and Keskinocak 2003). However, when consumers are forward-looking, responsive pricing may also have adverse effects owing to the interplay between the product's price path and consumers' adoption decisions, as epitomized by the well-known Coase conjecture (Coase 1972). In fact, the general consensus in the literature is that a firm facing strategic consumers will prefer a preannounced policy (see Cachon and Swinney 2009 for a notable exception). In a multiperiod fixed-quantity setting, Dasu and Tong (2010) provide an upper bound for expected revenues under preannounced and responsive pricing schemes and observe that a preannounced price plan with a small number of price changes performs nearly optimally. In a newsvendor model with strategic consumers, Su and Zhang (2008) argue that an endogenous salvage price (i.e., responsive pricing) amplifies consumers' incentive to delay their purchase until the salvage period. Aviv and Pazgal (2008) compare preannounced and responsive discounts under a more

[3] We note that cases where consumers can be more forward-looking than the firm are not considered in this paper.

[4] See Netessine and Tang (2009) for a comprehensive overview of operational strategies for managing strategic consumer behavior.

detailed consumer arrival process and find that the firm typically prefers preannounced pricing. Even in settings characterized by demand uncertainty where responsive pricing allows the firm to react optimally to updated demand information, Aviv et al. (2013) observe that strategic consumer behavior tends to render responsive pricing suboptimal. In the aforementioned papers, it is assumed that consumers are socially isolated, or, equivalently, that no reason exists for consumer interactions to be relevant (e.g., there is no quality uncertainty). The model we develop agrees with the consensus (i.e., that preannounced pricing is optimal) for the benchmark case in which the firm and the consumers operate in the absence of SL. Interestingly, we find that the equation changes when SL is accounted for: in the majority of cases, the firm's preference is reversed from a preannounced price plan (in the absence of SL) to a responsive price plan (in the presence of SL).

Apart from its contribution to the strategic consumer literature, this paper also adds to a growing stream of literature on "social operations management," which studies the implications of social interactions among consumers for firms' operational strategies. Hu et al. (2016) consider a firm selling two substitutable products to a stream of consumers who arrive sequentially and whose purchasing decisions can be influenced by earlier purchases. Candogan et al. (2012) and Hu and Wang (2013) study optimal pricing in social networks with positive externalities. Tereyağoğlu and Veeraraghavan (2012) consider a setting where consumers may use their purchases to display their social status. The type of social interaction considered in this paper is different: here, consumers interact with each other through product reviews with the goal of learning the unobservable quality of a new product; in this respect, our work connects to the SL literature, which we discuss next.

In the SL literature, customers are usually assumed to arrive at the firm sequentially and make once-and-for-all purchasing decisions; in other words, this work typically does not account for strategic consumer behavior. The seminal papers by Banerjee (1992) and Bikhchandani et al. (1992) illustrate that when the actions (e.g., adoption decisions) of the first few agents (e.g., consumers) reveal their private information regarding some unobservable state of the world (e.g., product quality), subsequent consumers may disregard their own private information and simply mimic the decision of their predecessor. Bose et al. (2008) illustrate how a monopolist employing dynamic pricing can use its pricing decision to control the amount of information that can be inferred by future consumers from the purchasing decision of the current consumer. Perhaps more relevant to the post-Internet era are models where SL occurs on the basis of reviews that reveal ex post consumer experiences, rather than actions that reveal ex ante private information. Ifrach et al. (2015) study monopoly pricing when consumers report whether their ex post derived utility was positive or negative. Papanastasiou et al. (2015) focus on the implications of SL on the quantity released by a monopolist during a new product's launch phase. Bergemann and Välimäki (1997) analyze the diffusion of a new product in a duopolistic market where the firm and the consumers learn the product's unknown value from the experiences of previous product adopters. Importantly, the above work does not account for the fact that consumers may initially decide not to purchase the product for strategic reasons (i.e., to gain information from product reviews), knowing that they can revisit their decision at a later point in time. By contrast, a recent paper by Yu et al. (2016) allows for such consumer behavior. Although their approach to modeling the SL process differs from ours (see Footnote 8), the two papers complement each other on many levels. Whereas Yu et al. (2016) consider responsive price plans exclusively, we analyze both preannounced and responsive price plans to perform a direct comparison between the two. Moreover, the analysis of responsive pricing in the current paper focuses more on the structure of equilibrium price paths, while Yu et al. (2016) emphasize the added value of SL for the firm and the consumers.

Finally, we note that consumers in our model face uncertainty regarding the intrinsic quality of a new and innovative product, but are fully informed about their idiosyncratic preferences. In other settings, consumers may initially be uninformed about their idiosyncratic preferences for a specific product, and learn these preferences over time (e.g., a traveler may initially be uncertain about his preferences for a ticket on a specific date of travel, but become informed as the date approaches). Since each consumer's preferences may depend on various exogenous factors, extant work has often assumed that this type of uncertainty is resolved exogenously in time. DeGraba (1995) demonstrates that a monopolist may use supply shortages to induce a buying frenzy among uninformed consumers (see also Courty and Nasiry 2016 for a dynamic model of frenzies). Swinney (2011) finds that when consumers learn their preferences over time, the value of quick-response production practices is generally diminished as a result of forward-looking consumer behavior. Prasad et al. (2011) investigate whether and how retailers should employ advance selling to uninformed consumers. An exception to the exogenous revelation of preferences assumed in the aforementioned papers can be found in the work of Jing (2011), who considers a responsive pricing problem where consumers are more likely to become informed about their preferences as the number of early adopters increases. In contrast to the above settings, where consumers face uncertainty about their own preferences (i.e., there is "one-sided" learning on attributes that are valued differently by each consumer), in the setting we consider, both the firm and the consumers face uncertainty about a new product's quality (i.e., there is "two-sided" learning on product attributes that are valued equally by all consumers).

3. Model Description

We consider a single firm selling a new product of ex ante unknown quality over two periods. The market consists of a continuum of consumers with total mass normalized to one, and each customer demands at most one unit of the product during the course of the selling season. Customer i's gross utility from purchasing the product comprises two components: a *preference* component, x_i, and a *quality* component, q_i (e.g., Villas-Boas 2004, Li and Hitt 2008). The value of the preference component x_i reflects the customer's idiosyncratic preferences over the product's ex ante observable attributes (e.g., brand, color). We assume that preference components, x_i, are distributed in the population according to the uniform distribution $U[0, 1]$. (The uniform assumption has no significant bearing on our results, but simplifies analysis and exposition.) The quality component q_i represents the product's quality for customer i, which is ex ante unknown; customers learn the value of q_i only after they purchase and experience the product. We assume that the distribution of ex post quality perceptions in the population is Normal, $q_i \sim \mathrm{N}(\hat{q}, \sigma_q^2)$, where $\hat{q}$ is the product's *unobservable* mean quality (henceforth referred to simply as product quality), and σ_q captures the degree of heterogeneity in postpurchase quality perceptions (relatively more "niche" products are typically associated with larger σ_q; see Sun 2012).[5] The wealth-equivalent net benefit of purchasing the product for customer i in period t, $t \in \{1, 2\}$, is defined by $u_{it} = \delta_c^{t-1}(x_i + q_i - p_t)$, where p_t is the price of the product in period t, and $\delta_c \in [0, 1]$ is a discount factor that applies to second-period purchases. Parameter δ_c represents the opportunity cost of delaying adoption, but may also be interpreted as a measure of customers' patience and therefore as a measure of how "strategic" consumers are (Cachon and Swinney 2009). Throughout our analysis, we say that customers are "myopic" when $\delta_c = 0$.

The product's unobservable quality, $\hat{q}$, is the object of social learning. We assume a symmetric informational structure between the firm and the consumers: both parties share a common and public prior belief over $\hat{q}$.[6] This belief is expressed in our model through the Normal random variable $\tilde{q}_p$, $\tilde{q}_p \sim \mathrm{N}(q_p, \sigma_p^2)$, where we fix $q_p = 0$ without loss of generality. All customers who purchase the product in the first period report their ex post derived product quality, q_i, to the rest of the market through product reviews (e.g., via an online review platform).[7] In the beginning of the second period, the firm and the consumers observe the reviews of first-period buyers and update their common belief over the product's mean quality from $\tilde{q}_p$ to $\tilde{q}_u$ according to Bayes' rule.[8] Specifically, if a mass of n_1 customers purchase and review the product in the first period, and the average rating of these reviews is R, then the posterior belief, $\tilde{q}_u$, is normally distributed, $\tilde{q}_u \sim \mathrm{N}(q_u, \sigma_u^2)$, with mean

$$q_u = \frac{n_1 \gamma}{n_1 \gamma + 1} R, \quad \text{where } \gamma = \frac{\sigma_p^2}{\sigma_q^2} \tag{1}$$

(e.g., see DeGroot 2005, Section 9.5; the variance of the posterior belief is given by $\sigma_u^2 = \sigma_p^2/(n_1\gamma + 1)$). The posterior mean q_u is a weighted average between the prior mean $q_p = 0$ and the average rating from first-period reviews R. The weight placed by consumers on R increases with the mass of reviews n_1 (henceforth referred to more naturally as the "number of reviews") and with the ratio γ.[9] Intuitively, a larger number

[5] We assume that an individual customer's x_i and q_i components are conditionally independent for simplicity in exposition. Such dependence can be incorporated in our model without changing our model insights: ex post quality perceptions (and therefore product reviews) will be biased by the idiosyncratic preferences of the reviewers; however, rational Bayesian consumers can readily account for this bias provided knowledge of the distribution of preferences x_i (see Papanastasiou et al. 2015).

[6] Since the firm and consumers hold the same prior belief, firm actions in our model cannot convey any additional information on product quality to the consumers (i.e., there is no scope for signaling); this informational structure is commonly assumed in the SL literature to focus attention on the peer-to-peer learning process (e.g., Bergemann and Välimäki 1997, Bose et al. 2008). Furthermore, although we do not model expert/critic reviews explicitly, these may take part in forming the public prior belief; Dellarocas et al. (2007) find that there is generally little overlap between the informational content of critic reviews and that of consumer reviews.

[7] It makes no difference in our model whether consumers report directly on quality or net or gross utility. To see why, note that the product's price history and the distribution of preferences in the population are common knowledge. Therefore, rational consumers can still employ (1) to learn product quality (as explained subsequently in the main text), albeit with a simple adjustment performed on the observed average rating R. Furthermore, we may also assume that only a fraction of first-period buyers produce reviews; this has no qualitative bearing on our model insights.

[8] For an alternative approach to modeling the SL process, see Bergemann and Välimäki (1997) and Yu et al. (2016). In that approach, first-period purchases are assumed to result in a single *aggregate* review signal, whose density function is specified by the modeler. Although that approach has qualitatively similar properties to the one used in our analysis, the processes by which reviews are generated by consumers and then aggregated into a single signal are left abstract. By contrast, our model has transparent microfoundations: consumers who purchase simply report their own derived quality, and consumers remaining in the market learn directly from these reports.

[9] Note that the normalization of the total mass of consumers to one is inconsequential: for the subsequent analysis to hold for a general

of reviews renders the average rating more credible. The ratio γ is a measure of the degree of ex ante quality uncertainty relative to the uncertainty (noise) in individual product reviews. Notice that when $\gamma = 0$, the SL process is essentially inactive: the posterior belief, $\tilde{q}_u$, is identical to the prior belief, $\tilde{q}_p$. This case reflects situations in which SL is either (i) irrelevant, because there is no ex ante quality uncertainty (i.e., $\sigma_p \to 0$) and therefore nothing to be learned from product reviews, or (ii) useless, because buyer reviews carry no useful information on product quality for future consumers (i.e., $\sigma_q \to +\infty$). At the other extreme, when $\gamma \to +\infty$, the SL process dominates the posterior belief: any positive number of buyer reviews causes the firm and consumers to completely abandon their prior. Throughout our analysis, we refer to γ as the *SL influence parameter*, since larger γ effectively means that the SL process is more influential in shaping the quality perceptions of future consumers.

All of the aforementioned are common knowledge. In addition, each customer has private knowledge of her idiosyncratic preference component, x_i. In the beginning of the selling season, the firm announces either (a) both the first- and second-period prices, p_1 and p_2 (*preannounced* pricing), or (b) only the first-period price, p_1, with the second-period price p_2 to be set in the beginning of the second period (*responsive* pricing).[10] Consumers exhibit forward-looking behavior: they observe the firm's announcement and purchase the product in the first period only if the following two conditions hold simultaneously: (i) their expected utility from purchase in the first period is nonnegative and (ii) their expected utility from purchase in the first period is not lower than the expected utility of delaying their purchasing decision. Any customers remaining in the market in the second period purchase a unit provided their expected utility from doing so is nonnegative. The firm seeks to maximize its overall expected profit. For simplicity, in our analysis we assume a firm discount factor of $\delta_f = 1$; our model insights hold qualitatively for any $\delta_f \geq \delta_c$. Furthermore, we assume that the firm operates in the absence of any binding capacity constraints and incurs a constant cost of c per customer served. We focus our analysis on cases of $c \in [0, 1)$ so that at least some customers have an ex ante valuation for the product that is higher than the product's production cost; our main results can be shown to hold also for cases of $c \geq 1$.

mass M of consumers, simply redefine γ as $\gamma = M(\sigma_p^2/\sigma_q^2)$ and consider n_1 to represent the proportion of the market that purchases in the first period.

[10] It is beyond the scope of our analysis to model how the firm credibly commits to prices; rather, our goal is to investigate the relative merits of committing to a price path, assuming that such a commitment is feasible (e.g., through repeated interactions with consumers; see Gilbert and Klemperer 2000, Liu and van Ryzin 2008).

4. A Rational Belief over Social Learning

In the second period of our model, the consumers observe the reviews of first-period buyers and use them to refine their belief over the product's quality, $\hat{q}$, from $\tilde{q}_p$ to $\tilde{q}_u$. If customer i remains in the market for the second period and the product's price is p_2, then she purchases the product only if $E[u_{i2}] = x_i + q_u - p_2 \geq 0$ (recall that q_u is the mean of the posterior belief $\tilde{q}_u$; see (1)). Now consider customer i's first-period decision. For the customer to make a decision on whether to purchase the product or delay her purchasing decision, she must form a rational belief over her second-period expected utility. In turn, to achieve the latter it is necessary for her to form a rational belief over the posterior parameter q_u; that is, the posterior mean q_u is viewed in the first period as a random variable, which is realized after the reviews of first-period buyers have been observed by the customer. This rational belief, termed the "preposterior" distribution of q_u, is described in Lemma 1.

Lemma 1. *Suppose that n_1 product reviews are available to customers remaining in the market in the second period. Then the preposterior distribution of q_u has a Normal density function with mean zero (i.e., equal to q_p) and standard deviation $\sigma_p\sqrt{n_1\gamma/(n_1\gamma+1)}$.*

All proofs are provided in the appendix. Ex ante, product reviews have no effect, on average, on the mean of customers' quality belief. The standard deviation of the preposterior distribution (which measures the extent to which the posterior mean is likely to depart from the prior mean) depends on the amount of information made available to the customer through product reviews and includes uncertainty regarding both the product's quality, $\hat{q}$, as well as the noise in individual buyers' product reviews. Perhaps counterintuitively, as the number of reviews increases and the information conveyed through these reviews becomes more precise, the variance of the preposterior distribution increases. To see why this is the case, note that if $n_1 = 0$, then no additional information is available in the second period, and customers' posterior mean, q_u, is exactly equal to the prior mean, i.e., $q_u = q_p = 0$. On the other hand, as the number of reviews increases, the posterior mean is likely to depart further from the prior mean, consistent with the preposterior distribution having greater variability.

Importantly, in the analysis that follows, the number n_1 of reviews generated in the first period will be an equilibrium outcome, because it depends directly on customers' first-period adoption decisions, which in turn depend on the firm's pricing policy. To conclude this section, we introduce the following notation, which will facilitate exposition of our results.

Definition 1. The probability density function $f(\cdot; z)$ corresponds to a zero-mean Normal random variable of standard deviation $\sigma(z)$, where $\sigma(z) := \sigma_p\sqrt{(1-z)\gamma/((1-z)\gamma+1)}$. Define also $F(\cdot; z)$ as the corresponding cumulative distribution function, and let $\bar{F}(\cdot; z) := 1 - F(\cdot; z)$.

5. Preannounced Pricing

We discuss first the pricing-adoption game when the firm employs a preannounced pricing policy. In the first period, the firm announces the full price path $\{p_1, p_2\}$. Customers take this announcement as given and make first-period purchasing decisions. In the second period, customers remaining in the market observe the reviews of the first-period buyers, update their beliefs over product quality, and make second-period purchasing decisions. Throughout our analysis, we focus on equilibria in pure strategies.

5.1. Benchmark: Preannounced Pricing Without Social Learning

It is instructive to begin with a brief description of the interaction between the firm and the consumers in the absence of SL (i.e., when product quality is known ex ante and/or product reviews are completely uninformative and/or consumers have no access to product reviews). A thorough analysis of preannounced pricing without SL can be found in the existing literature (e.g., Landsberger and Meilijson 1985) and is presented here in our model's notation for completeness. In our general setup, the absence of SL is captured by the limit case $\gamma \to 0$.[11]

When there is no SL, each customer takes the preannounced price plan $\{p_1, p_2\}$ as given and times her purchasing decision to maximize her utility. Given any arbitrary price plan, it is straightforward to deduce that consumer i will purchase the product in the first period provided $x_i \geq \tau(p_1, p_2)$, where

$$\tau(p_1, p_2) = \begin{cases} p_1 & \text{if } p_1 \leq p_2, \\ \dfrac{p_1 - \delta_c p_2}{1 - \delta_c} & \text{if } p_1 > p_2 \text{ and } p_1 - \delta_c p_2 \leq 1 - \delta_c, \\ 1 & \text{if } p_1 > p_2 \text{ and } p_1 - \delta_c p_2 > 1 - \delta_c. \end{cases} \quad (2)$$

Thus, when the product has an increasing or constant price plan ($p_1 \leq p_2$), any customer with nonnegative utility purchases in the first period. On the other hand, when the price is decreasing ($p_1 > p_2$), either (i) a positive number of high-valuation consumers purchase in the first period, despite the lower second-period price, to avoid discounted second-period utility (case $p_1 - \delta_c p_2 \leq 1 - \delta_c$), or (ii) no customers purchase in the first period because the second-period price is significantly lower than the first-period price (case $p_1 - \delta_c p_2 > 1 - \delta_c$). Furthermore, if customer i does not purchase in the first period, then she purchases in the second period provided $x_i \geq p_2$.

Given knowledge of the consumers' response to any arbitrary price plan, the firm chooses $\{p_1^*, p_2^*\}$ to maximize its overall profit, given by $\pi_{bp}(p_1, p_2) = (p_1 - c)[1 - \tau(p_1, p_2)]^+ + (p_2 - c)[\tau(p_1, p_2) - p_2]^+$, where we have used the notation $[r]^+ = \max[r, 0]$. The firm's optimal pricing policy is as follows.

Proposition 1. *In the absence of SL, any preannounced price plan generates a unique equilibrium in the pricing-adoption game. The firm's unique optimal policy is*

$$p_1^* = \frac{c(1+\delta_c)+2}{\delta_c+3} \quad \textit{and} \quad p_2^* = \frac{2c+\delta_c+1}{\delta_c+3}.$$

Furthermore, p_1^* (p_2^*) is decreasing (increasing) in δ_c, and the firm's profit $\pi_{bp}(p_1^*, p_2^*)$ is decreasing in δ_c.

Note that in the absence of SL, (i) the firm always announces a decreasing price plan (i.e., $p_1^* \geq p_2^*$); (ii) as customers become more patient, prices p_1^* and p_2^* approach each other; and (iii) as customers become more patient, firm profit decreases.

5.2. Preannounced Pricing with Social Learning

Let us now return to the general model, where consumers interact socially through product reviews to learn about the product's unknown quality. We first discuss the consumers' response to any arbitrary preannounced price plan. We then analyze the firm's pricing problem.

5.2.1. Consumers' Purchasing Strategy. How does the introduction of SL in the above benchmark model affect the consumers' purchasing strategy for a given price plan $\{p_1, p_2\}$? Consider how the actions of individual consumers affect the utility of their peers. In settings characterized by SL, information on product quality is both generated and consumed by the customer population. Each additional early purchase generates an additional product review, which in turn enables later customers to make an incrementally better-informed purchasing decision. In our model, an individual consumer's expected utility from delaying her purchasing decision (until the second period) increases with the number of customers who choose to purchase the product early (in the first period; see Lemma 5 in the appendix).

Once the firm announces its pricing policy, the customers engage in a purchasing game with each other. The equilibrium strategy adopted by the consumers is one characterized by a form of free riding, since

[11] Use of the limit $\gamma \to 0$ is prompted by Definition 1, according to which $f(\cdot; z)$ is not well defined for the case $\gamma = 0$.

customers are enticed to wait for the information generated by others rather than experiment with the new product themselves. However, this tendency to delay is mediated by the endogenous generation of information: the larger the number of customers who strategically delay their purchase, the less well-informed future decisions will be. Lemma 2 describes the consumers' equilibrium purchasing strategy.

Lemma 2. *For any given preannounced price plan* $\{p_1, p_2\}$, *there exists a unique equilibrium in the purchasing game played between the consumers. Specifically:*

(i) *In the first period, customer i purchases the product if* $x_i \geq \theta(p_1, p_2)$, *where*

$$\theta(p_1, p_2) = \begin{cases} y & \text{if } p_1 - \delta_c p_2 \leq 1 - \delta_c, \\ 1 & \text{if } p_1 - \delta_c p_2 > 1 - \delta_c, \end{cases}$$

and $y \in [p_1, 1]$ *is the unique solution to the implicit equation*

$$y - p_1 = \delta_c \int_{p_2 - y}^{\infty} (y + q_u - p_2) f(q_u; y)\, dq_u. \tag{3}$$

The threshold $\theta(p_1, p_2)$ *is increasing in* γ, p_1, *and* δ_c, *and decreasing in* p_2.

(ii) *In the second period, customer i purchases the product if* $p_2 - q_u \leq x_i < \theta(p_1, p_2)$, *where* q_u *is the realized posterior mean belief over quality.*

When the first-period price is significantly higher than the second-period price ($p_1 - \delta_c p_2 > 1 - \delta_c$), we observe "adoption inertia": the significant price benefit associated with second-period purchases makes all customers choose to defer their purchasing decision, even though second-period decisions will be made without any additional information from product reviews (since no sales occur in the first period). On the contrary, when p_1 is not much higher than p_2, a positive number of customers purchase the product in the first period. The left-hand side (LHS) of (3) represents the customer's first-period expected utility from purchase, whereas the right-hand side (RHS) represents her expected utility from delaying the purchasing decision. The lower limit of the integral accounts for the fact that, after observing the reviews of her peers, the customer will only purchase the product if her updated expected utility is positive—delaying the purchasing decision grants customers the right, but not the obligation, to purchase in the second period.

With regard to its dependence on p_1, p_2, and δ_c, the customers' purchasing strategy exhibits the intuitive properties that are also observed in the absence of SL. More important for the purposes of our analysis is the property pertaining to the SL influence parameter γ, which suggests that the first-period purchasing threshold becomes higher as SL becomes more influential—the obvious implication is that SL renders consumers "more strategic," in the sense that a larger number of strategic purchasing delays occur in its presence.

5.2.2. Firm's Pricing Policy and Profit. For the firm, optimizing the preannounced price plan is a convoluted task owing to the interaction between its pricing decisions, the adoption decisions of the strategic consumers, and the ex ante uncertain effects of the SL process on the valuations of second-period consumers. The analysis of this section is centered around two main questions. The first pertains to the optimal preannounced price plan: how should the firm adjust its pricing decisions to accommodate the SL process when dealing with strategic consumers? The second question concerns the firm's equilibrium payoff: given that SL exacerbates strategic consumer behavior (Lemma 2), is its presence beneficial or detrimental for the firm? Propositions 2 and 3, along with the discussions that follow them, address each question, respectively.

Given knowledge of customers' response to any arbitrary preannounced price plan, the firm chooses $\{p_1^*, p_2^*\}$ to maximize its expected profit, defined by

$$\begin{aligned} \pi_p(p_1, p_2) = {} & (p_1 - c)(1 - \theta) \\ & + (p_2 - c)\left(\int_{p_2 - \theta}^{p_2} [\theta + q_u - p_2] f(q_u; \theta)\, dq_u \right. \\ & \left. + \int_{p_2}^{+\infty} \theta f(q_u; \theta)\, dq_u \right), \end{aligned} \tag{4}$$

where the dependence of the threshold θ on p_1 and p_2 has been suppressed to simplify notation. The first and second terms correspond to first- and second-period profit, respectively. Whereas first-period profit is deterministic, the firm's second-period profit is ex ante uncertain owing to the demand uncertainty generated by the SL process—depending on the realization of the posterior parameter q_u, either none (low q_u) or a fraction (moderate q_u) or all (high q_u) of the remaining customers purchase the product in the second period.

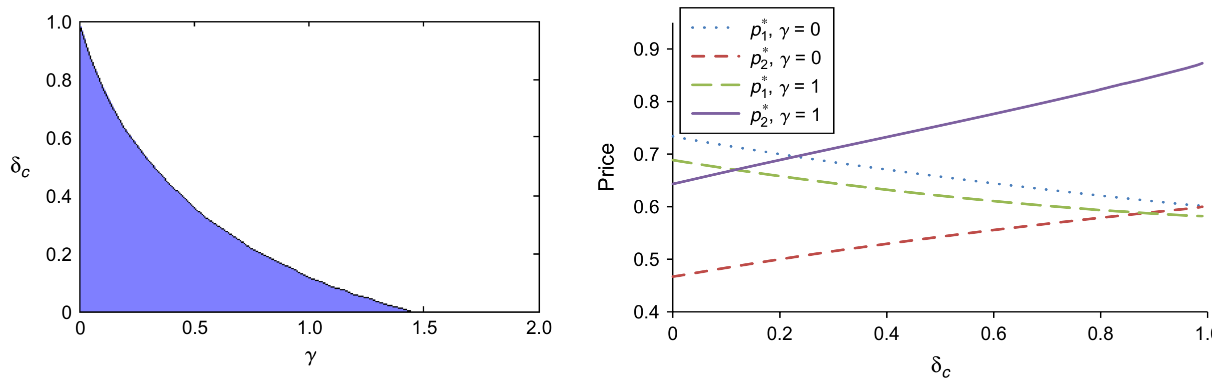
In terms of characterizing the firm's optimal price plan, the optimality conditions of problem (4) are, unfortunately, not very informative. We will first derive analytically the main properties of the optimal price plan and discuss their implications. We will then illustrate in more detail, through controlled examples, the mechanics underlying the firm's pricing decisions in the presence of SL and how these combine to form the optimal price plan.

Proposition 2. *In the presence of SL, any preannounced price plan generates a unique equilibrium in the pricing-adoption game. Furthermore:*

(i) *It can never be optimal for the firm to choose a price plan that induces adoption inertia in the first period; that is,* $p_1^* - \delta_c p_2^* \leq 1 - \delta_c$.

(ii) *There exists a threshold* $\Delta(\gamma)$ *such that if* $\delta_c \geq \Delta(\gamma)$, *the optimal price plan satisfies* $p_1^* < p_2^*$.

Figure 1 (Color online) (Left) Region Plot for the Structure of Optimal Preannounced Pricing Policies; (Right) Optimal First- and Second-Period Prices With and Without SL



Notes. In the left panel, shaded (white) regions mark the optimality of decreasing (increasing) price plans. Default parameter values are $\gamma = 1$, $\sigma_p = 1$, and $c = 0.2$.

We first point out that uniqueness of the equilibrium under any arbitrary price plan is an immediate consequence of Lemma 2. With respect to the firm's optimal policy, the first point of Proposition 2 suggests that adoption inertia can never be an optimal outcome for the firm. Since adoption inertia prohibits the generation of product reviews, the significance of this result is to establish that the SL process will always be "active" in equilibrium.[12]

The second point of Proposition 2 suggests a striking difference between firm pricing in the presence and absence of SL. Recall that in the absence of SL, the firm always announces a decreasing price path, aimed at exercising price discrimination (Proposition 1). This intuitive form of pricing may no longer be optimal in the presence of SL, especially when the firm faces consumers that are highly strategic. Instead, the firm in this case announces an increasing price plan—the presence of SL results in a *reversal* of the structure of the optimal price plan. The region plot on the left-hand side of Figure 1 supplements the result of Proposition 2.

Observe that the firm employs a decreasing price path only if SL is not significantly influential and/or the consumers' discount factor is low. By contrast, in most cases the firm announces a lowered "introductory" price followed by a higher regular price. What we observe here is that it is optimal for the firm to preannounce a second-period *information premium*, that is, to charge consumers for the privilege of making a better-informed purchasing decision. This premium has two effects. First, it counterbalances consumers' increased willingness to wait under SL and shifts demand back to the first period. Second, from those consumers who choose to wait despite the high second-period price, the firm extracts high profit in cases of highly favorable SL scenarios. Crucially, notice that the first effect feeds forward and reinforces the second, in the sense that a larger number of first-period reviews (generated by shifting demand back to the first period) renders highly favorable SL scenarios ex ante more probable (by increasing the ex ante variability of q_u; see Lemma 1).[13]

Let us now take a more detailed look at the drivers that shape the optimal preannounced pricing policy. To do so, we decompose the overall impact of SL into its two main effects and consider the implications of each effect in turn.

1. The *behavioral* effect changes consumers' purchasing behavior in the first period: as γ increases, consumers' informational incentive to delay their purchase increases, resulting in a larger number of strategic purchasing delays.

2. The *informational* effect shifts the demand curve faced by the firm in the second period: depending on whether (and the extent to which) reviews are favorable or not, the firm faces a population of relatively higher or lower valuations for the product in the second period.

The impact of the behavioral effect (viewed in isolation) on the optimal price plan can be deduced by leveraging the analysis of the benchmark model in §5.1. In particular, since this effect essentially renders consumers more patient, Proposition 1 suggests that the behavioral effect pushes p_1^* down and p_2^* up, toward each other.

The implications of the informational effect are less straightforward. This effect operates on the valuations

[12] We note that a preannounced market exit (e.g., announcing $\{p_1, p_2\}$ with $p_2 \to +\infty$) is profit-equivalent to adoption inertia, and is therefore also strictly suboptimal for the firm. To see why, note that both strategies confine sales to occur in a single period and under no information from product reviews.

[13] Swinney (2011) illustrates that when consumers' preferences are revealed exogenously over time (as opposed to product quality being learned endogenously through SL), it is optimal for the firm to employ an increasing price plan to decrease strategic purchasing delays among uninformed consumers. The increasing price plan in our model also reduces strategic delays, but, importantly, it also serves the purpose of reinforcing the firm's second-period profit in high-quality scenarios.

of second-period consumers, and as such has a significant impact on the firm's preannounced second-period price. To illustrate, we construct a paradigm in which the informational effect is active, but the behavioral effect is "switched off" by making customers myopic.

Example 1. Suppose that $\delta_c = 0$, and fix the first-period price at some arbitrary $p_1 > 0$. Then, for any $k > 0$, $p_2^*|_{\gamma=k} > p_2^*|_{\gamma\to 0}$.

Thus, all else being equal, the firm chooses a higher second-period price in the presence of SL. The rationale here is based on the symmetric nature of the uncertainty faced by the firm (q_u is an ex ante Normal random variable; see Lemma 1). For every favorable SL scenario (there exists a continuum of these), there exists a corresponding unfavorable "mirror" scenario that is equally probable. By announcing a higher second-period price, the firm is able to capitalize on highly favorable scenarios more effectively, while at the same time its profit in the corresponding highly unfavorable scenarios is at worst zero.

The combined impact of the behavioral and informational effects on the optimal price plan is illustrated on the right-hand side of Figure 1. The behavioral effect causes a decrease in p_1^* and an increase in p_2^*, while the informational effect causes a further increase in p_2^*. As suggested by Proposition 2, unless δ_c is low, the two effects combined are significant enough to result in a reversal of the optimal price plan from decreasing to increasing.

We next address our second question, regarding the firm's profit in the presence of SL. Recall that Lemma 2 establishes that the presence of SL renders consumers more strategic. Even though the firm can do its best to mitigate the negative effects of SL on strategic consumer behavior through its pricing policy, it is unclear whether the overall impact of SL on expected firm profit is positive or negative; Proposition 3 makes progress in answering this question.

Proposition 3. *In the presence of SL, there exist thresholds $\Delta_{lp}(\gamma) \in (0, 1]$ and $\Delta_{hp}(\gamma) \in [0, 1)$ such that if $\delta_c \le \Delta_{lp}(\gamma)$ or $\delta_c \ge \Delta_{hp}(\gamma)$, then the firm achieves greater expected profit than it achieves in the absence of SL.*

The result that SL is beneficial for the firm, particularly for high values of δ_c, is surprising. In particular, notice that (i) SL renders consumers more strategic, and (ii) under preannounced pricing, the firm does not have the flexibility to adjust the product's price according to the content of first-period reviews. Thus, the SL process seemingly puts the firm at a double disadvantage.

To explain the intuition underlying Proposition 3, we again use the decomposition of SL into its two main effects, the behavioral and the informational, as described above. The behavioral effect results in a decrease in expected firm profit—this much is evident from Proposition 1, which suggests that as consumers become more patient, firm profit decreases. By contrast, the informational effect has a positive impact on the firm's expected profit; to illustrate, we present the following example, which isolates the informational effect by assuming myopic consumers.

Example 2. Suppose that $\delta_c = 0$. Then, for any $k > 0$, $\pi_p^*|_{\gamma=k} > \pi_p^*|_{\gamma\to 0}$.

That is, in the absence of the behavioral effect, it is always possible for the firm to identify a price plan that takes advantage of the probabilistic shift in consumers' second-period valuations to generate higher expected profit.

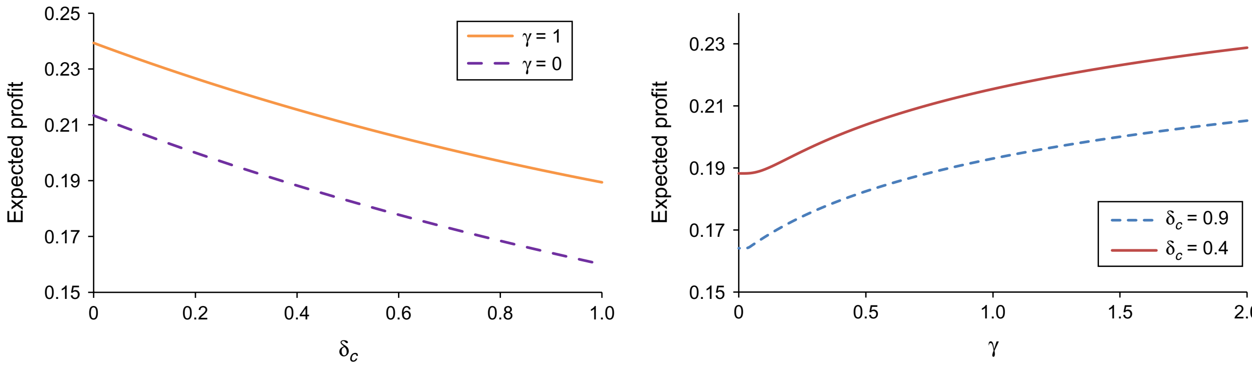
Whether the overall impact of SL on expected firm profit is positive or negative depends on the relative magnitude of the two opposing effects. When δ_c is low, the behavioral effect is weak and the beneficial informational effect results in an increase in expected firm profit. The more interesting case is that of high δ_c, where the negative behavioral effect is at its worst; Proposition 3 suggests that even when this is the case, the positive informational effect dominates, resulting in an increase in expected profit. Although the result of Proposition 3 admits the possibility that the presence of SL is detrimental for some intermediate values of δ_c, we were unable to find any such cases in our numerical experiments; Figure 2 is typical of our observations.

To conclude this section, we consider how the result of Proposition 3 is affected if we restrict the firm to charge a fixed price (i.e., by adding the constraint $p_1 = p_2$ to problem (4)). This issue is of particular relevance in settings where fairness considerations are important for long-term firm–customer relationships (e.g., Hafner and Stone 2007), or when implementing price changes is costly or impractical (see also Aviv and Pazgal 2008); in such settings, the firm may be reluctant to price intertemporally. As we show in the proof of Proposition 3, the result continues to hold unchanged for the case of fixed pricing.

6. Responsive Pricing

Now suppose that the firm does not commit ex ante to a full price path. Under a responsive pricing policy, the game between the firm and the consumers is modified as follows: In the beginning of the first period, the firm sets the first-period price, p_1, and consumers make first-period purchasing decisions. In the beginning of the second-period, the firm and consumers observe the product reviews generated by first-period buyers and update their beliefs over product quality. The firm then sets the second-period price, p_2, and consumers remaining in the market make second-period purchasing decisions. The two-period stochastic game between the firm and the consumers

Figure 2 (Color online) Optimal Preannounced Profit at Different Combinations of SL Influence (γ) and Consumer Patience (δ_c)



Note. Default parameter values are $\sigma_p = 1$ and $c = 0.2$.

is analyzed in reverse chronological order; we seek pure-strategy subgame-perfect equilibria.

6.1. Benchmark: Responsive Pricing Without Social Learning

We begin, as in §5, with a brief discussion of the benchmark case where there is no SL ($\gamma \to 0$). For a more thorough analysis of responsive pricing with strategic consumers see, for example, Besanko and Winston (1990).

Consider first the second-period subgame. Because for any first-period price p_1 the consumers adopt a threshold purchasing policy in the first period (Besanko and Winston 1990), consumers remaining in the market in the second period have total mass $\hat{x}$ and idiosyncratic preference components x_i distributed uniformly $U[0, \hat{x}]$, for some $\hat{x} \in [0, 1]$. The firm chooses the second-period price p_2 to maximize $\pi_{br2}(p_2) = (p_2 - c)(\hat{x} - p_2)$. Thus, the firm charges $p_2^* = (\hat{x} + c)/2$, and consumers purchase provided their expected utility is nonnegative. Given any $\hat{x}$, the equilibrium in the second-period subgame is unique.

In the first period, the firm and consumers anticipate the effects of their actions on the equilibrium of the second-period subgame. Given a first-period price p_1, consumer i forms beliefs (which are correct in equilibrium) over $\hat{x}$ and $p_2^*(\hat{x})$ and purchases only if (i) $E[u_{i1}] = x_i - p_1 \geq 0$ and (ii) $E[u_{i1}] \geq \delta_c(x_i - p_2^*(\hat{x})) = E[u_{i2}]$. Consequentially, the unique optimal purchasing strategy for the strategic consumers is to purchase in the first period only if $x_i \geq \chi(p_1)$, where

$$\chi(p_1) = \begin{cases} \dfrac{2p_1 - c\delta_c}{2 - \delta_c} & \text{if } p_1 \leq \dfrac{2 - \delta_c(1-c)}{2}, \\[2ex] 1 & \text{if } p_1 > \dfrac{2 - \delta_c(1-c)}{2}. \end{cases} \tag{5}$$

When the product's introductory price is too high ($p_1 > (2 - \delta_c(1-c))/2$), all customers prefer to delay their purchase until the second period, expecting that the firm will lower the price significantly. When this is not the case ($p_1 \leq (2 - \delta_c(1-c))/2$), higher-expected-utility customers purchase in the first period, while lower-expected-utility customers prefer to defer their purchase.

At the beginning of the game, the firm, anticipating customers' first-period response to any arbitrary price p_1, as well as the outcome of the second-period subgame, chooses the introductory price p_1^* that maximizes its overall profit, which may be expressed as $\pi_{br}(p_1) = (p_1 - c)(1 - \chi(p_1)) + (\chi(p_1) - c)^2/4$. The full equilibrium price path is described in Proposition 4.

Proposition 4. *In the absence of SL, any first-period price generates a unique equilibrium in the pricing-adoption game. The firm's unique optimal policy is*

$$p_1^* = \frac{2c + \delta_c^2(1-c) + 4(1-\delta_c)}{6 - 4\delta_c} \quad \textit{and} \quad p_2^* = \frac{\chi(p_1^*) + c}{2}.$$

Furthermore, p_1^* (p_2^*) is decreasing (increasing) in δ_c, and the firm's profit $\pi_{br}(p_1^*)$ is decreasing in δ_c.

Similarly as in the case of preannounced pricing, (i) the equilibrium price path is always decreasing (i.e., $p_1^* \geq p_2^*$); (ii) as consumers become more patient, prices p_1^* and p_2^* approach each other; and (iii) as consumer become more patient, firm profit decreases.

6.2. Responsive Pricing with Social Learning

We now return to the general model, where the SL process is influential. We analyze first the equilibrium of the second-period subgame. We then consider the consumers' first-period purchasing strategy and examine the implications of SL for the firm's pricing policy and profit.

6.2.1. Second-Period Subgame. To analyze the second-period subgame, we temporarily assume that in the first period, for any first-period price p_1 chosen by the firm, customers adopt a threshold purchasing policy—the validity of this assumption is proven in the next section. In the beginning of the second period and as a result of customers' first-period purchasing decisions, the firm faces a population of consumers of total mass $\bar{x}$ with idiosyncratic preference components x_i distributed uniformly $U[0, \bar{x}]$, for some $\bar{x} \in [0, 1]$.

In the presence of SL, the interaction between the firm and consumers in the second period is characterized by the influence of the informational effect. Consumers remaining in the market observe the reviews of first-period buyers and arrive at an updated willingness to pay, which, for customer i, is given by $x_i + q_u$. Thus, depending on the content of reviews, the firm in the second period faces a population that has a relatively higher or lower willingness to pay than in the first period. The firm's profit, as a function of its second-period pricing decision, is defined by

$$\pi_2(p_2) = (p_2 - c)[\min(\bar{x}, \bar{x} + q_u - p_2)]^+,$$

and the unique equilibrium of the second-period subgame is described in Lemma 3.

Lemma 3. *Under responsive pricing, given any q_u and $\bar{x}$, there exists a unique equilibrium in the second-period subgame played between the firm and the consumers. Specifically*:

(i) *The firm's optimal second-period pricing policy is defined by*

$$p_2^*(q_u, \bar{x}) = \begin{cases} c & \text{if } q_u \le c - \bar{x}, \\ \dfrac{q_u + c + \bar{x}}{2} & \text{if } c - \bar{x} < q_u \le c + \bar{x}, \\ q_u & \text{if } q_u > c + \bar{x}. \end{cases} \tag{6}$$

(ii) *Customer i purchases the product in the second period if $p_2^*(q_u, \bar{x}) - q_u \le x_i < \bar{x}$.*

Customers purchase the product provided their expected utility from purchase (given what they have learned from reviews and the firm's decision p_2^*) is nonnegative. The firm's profit-maximizing p_2 depends on the SL outcome q_u as well as customers' first-period purchasing decisions, which specify $\bar{x}$. If q_u is very low (a sign of low quality for the firm and consumers), the firm cannot extract positive profit at any price p_2, and therefore exits the market; this is signified by a second-period price of $p_2^* = c$ at which no purchases occur. If q_u is at intermediate levels, the firm chooses a price at which only a fraction of consumers remaining in the market choose to adopt the product. Finally, if q_u is very high, the firm finds it most profitable to choose the market-clearing price, $p_2^* = q_u$.

Note that q_u is an ex ante Normal random variable whose variance is increasing in γ. Since $p_2^*(q_u, \bar{x})$ is nondecreasing and convex in q_u, it follows (from properties of the Normal distribution) that given any $\bar{x}$, the expected second-period price is higher in the presence of SL ($\gamma > 0$) than it is in its absence ($\gamma \to 0$). Thus, in some sense, the impact of the informational effect on the second-period price under responsive pricing parallels that under preannounced pricing: in both cases, the informational effect leads to relatively increased prices in the second period.

6.2.2. Consumers' First-Period Purchasing Strategy. In the first period, the firm and consumers anticipate the effects of their actions on the equilibrium of the second-period subgame. However, since the realization of the posterior mean q_u is ex ante uncertain, the equilibrium in the second-period subgame is itself uncertain; unlike the benchmark case in §6.1, the firm and consumers in this case form rational *probabilistic* beliefs over the second-period equilibrium.

Consider the consumers' first-period purchasing strategy for any arbitrary price p_1. The consumers anticipate not only how their own opinions may change in the second period as a result of the available reviews, but also what the firm's reaction to these reviews will be—the informational advantage created by the availability of product reviews may presumably be absorbed, or even reversed, by the firm's second-period pricing flexibility. Lemma 4 characterizes the customers' first-period adoption decisions.

Lemma 4. *Under responsive pricing and given any first-period price p_1, there exists a unique optimal first-period purchasing strategy for the consumers. Specifically, customer i purchases the product in the first period if $x_i \ge \zeta(p_1)$, where*

$$\zeta(p_1) = \begin{cases} \psi & \text{if } p_1 \le \dfrac{2 - \delta_c(1-c)}{2}, \\ 1 & \text{if } p_1 > \dfrac{2 - \delta_c(1-c)}{2}, \end{cases}$$

and $\psi \in [p_1, 1]$ is the unique solution to the implicit equation

$$\psi - p_1 = \delta_c \int_{c-\psi}^{+\infty} (\psi + q_u - p_2^*(q_u, \psi)) f(q_u; \psi)\, dq_u, \tag{7}$$

with $p_2^*(q_u, \psi)$ specified in (6). The threshold $\zeta(p_1)$ is increasing in γ for any $c > 0$, increasing in p_1 and δ_c, and decreasing in c.

If the first-period price is too high ($p_1 > (2 - \delta_c(1-c))/2$), all consumers delay their purchasing decision (adoption inertia) in anticipation of a significantly lower second-period price. If the first-period price is not too high ($p_1 \le (2 - \delta_c(1-c))/2$), customers with relatively higher valuations purchase in the first period, while the rest of the population delays the purchasing decision. The final statement of the lemma highlights the behavioral effect of SL under responsive pricing: the more influential the SL process, the larger the number of strategic adoption delays.[14]

[14] In the special case $c = 0$, the consumers' first-period adoption strategy is independent of γ (see proof of Lemma 4).

6.2.3. Firm's Pricing Policy and Profit. We now bring the preceding analyses together and consider the implications of SL for the product's equilibrium price path and the firm's expected profit; these are addressed in the discussions that follow Propositions 5 and 6, respectively.

In the first period, taking into account the consumers' response to any arbitrary p_1, as well as the probabilistic equilibrium of the second-period subgame that results, the firm chooses p_1^* to maximize its overall expected profit,

$$\pi_r(p_1) = (p_1 - c)(1-\zeta) + \int_{c-\zeta}^{c+\zeta} \left(\frac{q_u + \zeta - c}{2}\right)^2 f(q_u;\zeta)\, dq_u + \int_{c+\zeta}^{+\infty} \zeta q_u f(q_u;\zeta)\, dq_u, \tag{8}$$

where the dependence of the threshold ζ on p_1 has been suppressed. As opposed to the case of preannounced pricing, problem (8) accounts for the fact that in the second period, the firm will adjust the product's price in response to the content of product reviews.

The equilibrium price path, which consists of the first-period price that maximizes (8) and the second-period price that is adapted to the content of product reviews, is described as follows.

Proposition 5. *In the presence of SL, any first-period price generates a unique equilibrium in the pricing-adoption game. Furthermore*:

(i) *It can never be optimal for the firm to choose a first-period price that induces adoption inertia; that is,* $p_1^* \le (2 - \delta_c(1-c))/2$.

(ii) *The optimal second-period price is defined by*

$$p_2^* = \begin{cases} c & \text{if } q_u \le c - \zeta(p_1^*), \\ \dfrac{q_u + c + \zeta(p_1^*)}{2} & \text{if } c - \zeta(p_1^*) < q_u \le c + \zeta(p_1^*), \\ q_u & \text{if } q_u > c + \zeta(p_1^*), \end{cases}$$

where q_u is the realized posterior mean belief over quality, and $\zeta(p_1^*)$ is described in Lemma 4.

As in the case of preannounced pricing, adoption inertia is strictly suboptimal for the firm and never arises in equilibrium. Recall that in the absence of SL, Proposition 4 describes a price path that is ex ante deterministic and decreasing. By contrast, the price path described in Proposition 5 is ex ante stochastic, since it depends on the realization of the posterior mean q_u. Furthermore, because q_u is an ex ante Normal random variable for any $\gamma > 0$, both increasing and decreasing price paths occur with positive probability.

Let us now take a closer look at the implications of SL for the equilibrium price path. First, note that Lemma 4 reveals that consumers become more patient in the presence of SL. As a consequence, Proposition 4 suggests that this behavioral effect of SL causes the firm to lower the product's introductory price (e.g., see the left-hand side of Figure 3). The second-period price is ex ante stochastic and depends on the content of the reviews generated in the first period.[15] As discussed in §6.2.1, owing to the firm's adaptation to the informational effect, the expected value of the second-period price is increased (conditional on consumers' first-period decisions) with respect to the case in which SL is absent. If the two effects combined are strong enough (this occurs when δ_c and γ are high), what results is an equilibrium price path that (in expectation) is increasing over time; this phenomenon is illustrated in the right-hand region plot of Figure 3.

In our numerical experiments, we observe that equilibrium price paths tend to be decreasing (i.e., this occurs in the majority of parameter combinations considered). More specifically, increasing expected price paths occur under the following conditions: (i) consumers are highly patient, (ii) SL is very influential, and (iii) marginal cost is high with respect to consumers' prior valuations. These three conditions paint the picture of a new-to-the-world product that is introduced with a high level of quality uncertainty and is relatively costly to produce. In such scenarios, the firm introduces the product at a low price, with the prospect of extracting high revenues later in the season by capitalizing on the (hopefully favorable) early reviews. In scenarios where the three conditions mentioned above do not apply, the expected price path remains decreasing, but is "flatter" compared to that in the absence of SL (i.e., the first-period price is decreased and the expected second-period price is increased).

To conclude our discussion of responsive pricing, we analyze the implications of SL for expected firm profit. Lemma 4 suggests that consumers become more strategic in the presence of SL, and Proposition 4 suggests that this behavioral effect, viewed in isolation, has a detrimental impact on expected firm profit. As was the case under preannounced pricing, the negative behavioral effect is opposed by the positive informational effect; this is illustrated in Example 3.

Example 3. Suppose $\delta_c = 0$. Then, for any $k > 0$, $\pi_r^*|_{\gamma=k} > \pi_r^*|_{\gamma\to 0}$.

It remains to establish whether, as in the case of preannounced pricing, the positive informational effect is sufficiently large to overpower the negative behavioral effect; in this respect, Proposition 6 mirrors the result of Proposition 3.

[15] It is common for the price of experiential products to change over time, especially in online settings (e.g., Vanek Smith 2012). Furthermore, anecdotal evidence suggests that the price of smartphone applications is positively correlated with their average review rating (Eberhardt 2014).

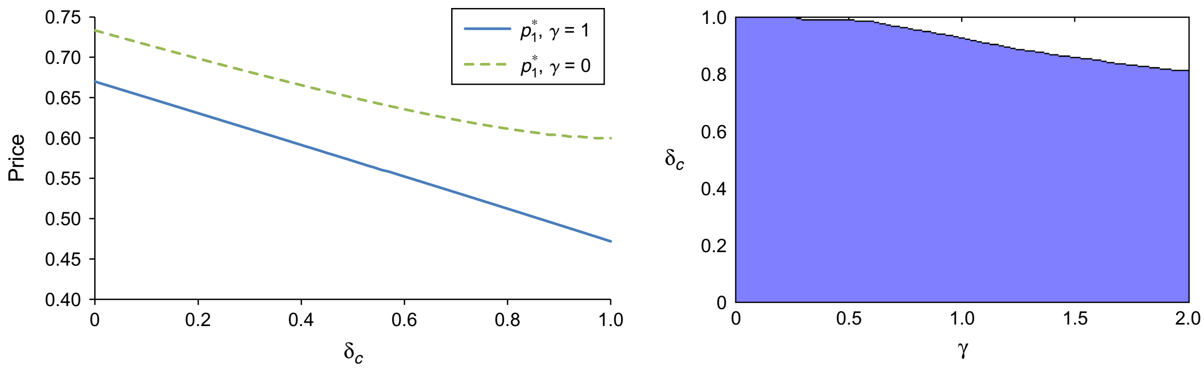


Figure 3 (Color online) (Left) Optimal Introductory Price With and Without SL; (Right) Region Plot for the Structure of Equilibrium Price Paths Under Responsive Pricing

Notes. In the right panel, shaded (white) regions mark price paths that are decreasing (increasing) in expectation. Default parameter values are $\gamma = 1$, $\sigma_p = 1$, and $c = 0.2$.

Proposition 6. *In the presence of SL, there exist thresholds $\Delta_{lr}(\gamma) \in (0, 1]$ and $\Delta_{hr}(\gamma) \in [0, 1)$ such that if $\delta_c \leq \Delta_{lr}(\gamma)$ or $\delta_c \geq \Delta_{hr}(\gamma)$, then the firm achieves greater expected profit than it achieves in the absence of SL.*

The beneficial effects of SL on expected profit when the firm adjusts prices dynamically have been established previously in the literature, but under the assumption that consumers are nonstrategic (e.g., Bose et al. 2008, Ifrach et al. 2015). Proposition 6 generalizes this finding to the case of forward-looking consumers, by establishing that the positive effects of SL remain dominant even once strategic consumer behavior is accounted for. This result proves the beneficial nature of the SL process only for low and high values of δ_c; however, our numerical experiments suggest that, as in the case of preannounced pricing, the result holds for all combinations of our model parameters.[16]

7. Preannounced vs. Responsive Pricing

A recurring theme in the recent literature that considers strategic consumer behavior is the value of price commitment for the firm. For instance, Aviv and Pazgal (2008) report that when customers are forward-looking (and in the absence of future rationing risk), preannounced pricing is a more effective way of extracting profit than responsive pricing (see "announced" and "contingent" pricing in their model).[17] In the benchmark where there is no SL ($\gamma \to 0$), our model replicates this prediction.

Proposition 7. *In the absence of SL, the firm's profit is higher under preannounced pricing than it is under responsive pricing.*

The question of interest in this section is whether price commitment is preferred by the firm when SL is influential. Interestingly, we observe that, in most cases, the *opposite* is true. Proposition 8 describes the nature of this observation, which is also illustrated graphically in Figure 4.

Proposition 8. *In the presence of SL, there exists a threshold $T(\gamma) \in (0, 1]$ such that if $\delta_c \leq T(\gamma)$, the firm's profit is higher under responsive pricing than it is under preannounced pricing.*

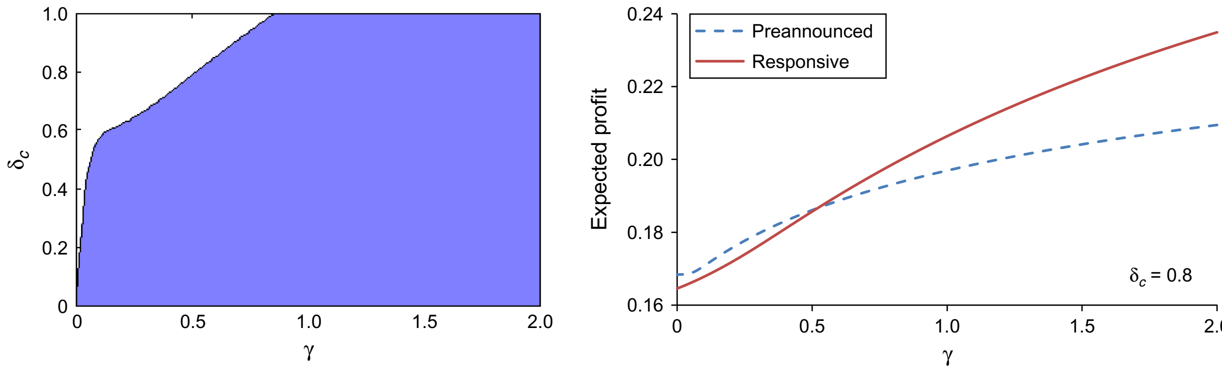
In the example of Figure 4, the threshold $T(\gamma)$ is equal to one for cases of moderate and high γ. (Across our experiments, the threshold $T(\gamma)$ does not change significantly for different values of c.) More generally, our numerical study points to three main observations: first, a responsive pricing policy is optimal in the vast majority of cases; second, the cases in which a preannounced price plan is preferred are those that combine patient customers with weak SL influence (i.e., high δ_c and low γ); third, in those cases where a preannounced price plan is optimal, the increase in profit with respect to the optimal responsive price plan is much smaller (1.6% on average) than the corresponding increase in profit when a responsive price plan is optimal (8.8% on average).

The rationale underlying these observations is as follows. The flexibility offered by a responsive pricing policy is significantly advantageous for the firm when the valuations of second-period consumers are likely to change significantly as a result of SL. Ceteris paribus, a significant change in consumer valuations occurs when the number of first-period reviews is large (high n_1) and/or when the influence of SL is strong (high γ). When the value of γ is low, the only way in which the

[16] We note that in our model the firm is, by assumption, more patient than the consumers (since it does not discount its second-period profit). When the opposite is true (i.e., the firm is less patient than the consumers), the analysis of Yu et al. (2016) suggests that in some cases the firm can be worse off in the presence of SL.

[17] This conclusion persists even in cases where early sales can be used to update demand information (e.g., Aviv et al. 2013).

Figure 4 (Color online) (Left) Region Plot for the Optimal Class of Dynamic Pricing Policies as a Function of Consumer Patience (δ_c) and SL Influence (γ); (Right) Expected Firm Profit Under the Optimal Preannounced and Responsive Pricing Policies as a Function of SL Influence (γ)



Notes. In the left panel, shaded (white) regions mark the optimality of a responsive (preannounced) pricing policy. Default parameter values are $\sigma_p = 1$, $\delta_c = 0.8$, and $c = 0.2$.

firm can profit substantially from its second-period flexibility is if it generates a very large number of first-period sales/reviews. As customers become progressively more patient (i.e., δ_c increases), the firm is required to introduce the product at a progressively lower price to achieve the volume of reviews necessary to capitalize on its flexibility. For this reason, when δ_c is high and γ is low, the firm prefers to employ the optimal preannounced price path, rather than the optimal responsive price plan (which would entail either a large amount of sales at a very low price, or reduced effectiveness of pricing flexibility). Moreover, we note that even when the firm does prefer a preannounced price plan, the advantage with respect to a responsive policy is small and quickly disappears as γ increases from zero (e.g., see the right-hand plot of Figure 4).[18]

Finally, consider the customers' perspective. In the absence of SL, the firm's profit is maximized under a preannounced price plan (Proposition 7), but it is straightforward to show that the opposite is true for consumer surplus (i.e., in the absence of SL, consumer surplus is higher under responsive pricing for any $\delta_c \in [0, 1]$). In our numerical study, we find that the consumers' preference for responsive pricing continues to hold for any $\gamma > 0$. Combining this observation with Proposition 8, we conclude that SL has the positive effect of aligning the preferences of the firm and consumers regarding which class of policies arises in equilibrium (this is true whenever responsive pricing is preferred by the firm). This has the further implication that, among the two classes of policies available to the firm, the class chosen is (in most cases) the one that maximizes expected total welfare (i.e., the sum of expected firm profit and expected consumer surplus).

[18] An interesting avenue for future research is the analysis of hybrid "preannounced responsive" price plans in the presence of SL. In this case, the second-period price would be a function of the content of first-period reviews, but this dependence would be preannounced. Presumably, such price plans may perform better than pure preannounced or responsive price plans, since they allow the firm to capitalize on the learning process, while at the same time containing the negative effects of strategic consumer behavior. However, we point out the significant operational challenges associated with implementation of such pricing policies.

8. Conclusion

This paper presents a stylized analysis of the effects of social learning on the strategic interactions between a dynamic-pricing monopolist and a population of strategic consumers.

Recent research has highlighted that firms may neglect the ever-increasing sophistication of the modern-day consumer at their peril (e.g., Aviv and Pazgal 2008). However, this research has neglected perhaps one of the most important aspects of this sophistication: the ability of consumers to exchange experiences and learn from their peers. This paper demonstrates that pricing techniques that have come to constitute conventional wisdom may in fact be overturned by the increasing influence of SL on consumer decision making. For instance, we have shown that price commitment, which has been advocated as an effective way of managing strategic consumer behavior, is in most cases suboptimal for the firm once SL is accounted for. This suggests that managers pricing products for which product reviews are known to be a significant driver of demand could benefit from pricing products dynamically in response to buyers' sentiments (e.g., media items on Amazon.com). Furthermore, this paradigm shift from preannounced to responsive pricing constitutes a "win–win" situation—both the firm and the consumer population benefit from responsive prices.

Another main result of our analysis is that, even taking into account the negative effects of strategic consumer behavior, the SL process is one that should be endorsed and promoted by modern-day firms (in our model, the SL influence parameter γ and expected firm profit are positively related under either class of pricing policies). There are at least three dimensions along which firms can act to enhance the influence of SL on consumers' product evaluations and purchasing decisions. The first is to simply encourage the consumer population to pay more attention to buyer opinions; for instance, KIA and Ford Motors recently invested heavily in advertising campaigns aimed at promoting consumer attention to buyer reviews (see the "Reviews and Recommendations" and "Good Reviews" campaigns, respectively). The second is to provide the platform upon which consumers can communicate and exchange their product experiences; examples of high-profile firms that have actively facilitated the exchange of consumer experiences on their online spaces include Amazon.com, Dell Computers, and Apple. A third dimension is to increase the precision of buyer-generated reviews by asking consumers to rate products on multiple dimensions (e.g., see reviews on Hotels.com) rather than simply providing a one-dimensional rating.

We also point out two more subtle implications of our work. The first is associated with firms' new product development strategies. Specifically, we find that in the presence of SL, greater ex ante quality uncertainty is associated with higher expected firm profit. This result suggests that firms developing products for which SL is known to be significantly influential (e.g., high-tech electronics) may be encouraged to take risks and innovate with their new products, as opposed to incrementally improving previous product versions: when the resulting product proves to be a success, the SL process is handsomely rewarding, whereas in cases of product failures, the corresponding downside is relatively less severe. Second, we have seen that the SL process generates a relative increase in strategic purchasing delays. This finding may have implications for researchers and firms attempting to estimate the effects of SL on firm profits (e.g., as in Moretti 2011). Specifically, neglecting to account for consumers' increased tendency to delay purchase may lead to overestimates of the impact of SL, since intertemporal demand shifts (i.e., from earlier to later in the selling season) may be misinterpreted as demand generated through the SL process.

Appendix

In all proofs that follow, we use ϕ (Φ) to denote the standard Normal probability density function (cumulative distribution function), with $\bar{\Phi}(\cdot) := 1 - \Phi(\cdot)$.

Proof of Lemma 1. In the second period, if the product reviews of n_1 first-period buyers have a mean of R, then, by Bayes' rule, the posterior belief over $\hat{q}$, denoted by $\tilde{q}_u$, is Normal with a mean of q_u, where

$$q_u = \frac{\sigma_q^2}{n_1\sigma_p^2 + \sigma_q^2} q_p + \frac{n_1\sigma_p^2}{n_1\sigma_p^2 + \sigma_q^2} R = \frac{1}{n_1\gamma + 1} q_p + \frac{n_1\gamma}{n_1\gamma + 1} R,$$

where q_p is the mean of the prior belief, and $\gamma = \sigma_p^2/\sigma_q^2$. In the first period, the posterior mean belief q_u is viewed as a random variable (r.v.), since it depends on the unobservable realization of product quality $\hat{q}$ as well as the noise in first period reviews (i.e., it is subject to sampling error). Specifically, if the product's quality realization is $\hat{q}$, the sample mean of n_1 (independent and identically distributed Normal) reviews, R, follows $R \sim \mathrm{N}(\hat{q}, \sigma_q^2/n_1)$ so that $q_u \mid \hat{q} \sim \mathrm{N}((\sigma_q^2/(n_1\sigma_p^2 + \sigma_q^2))q_p + (n_1\gamma/(n_1\gamma+1))\hat{q}, (n_1\gamma/(n_1\gamma+1))^2(\sigma_q^2/n_1))$. Therefore, since $\hat{q}$ is an ex ante Normal r.v., $\hat{q} \sim \mathrm{N}(q_p, \sigma_p^2)$, $E[q_u] = E(E[q_u \mid \hat{q}]) = E((\sigma_q^2/(n_1\sigma_p^2 + \sigma_q^2))q_p + (n_1\gamma/(n_1\gamma+1))\hat{q}) = q_p$, and $\mathrm{Var}[q_u] = E(\mathrm{Var}[q_u \mid \hat{q}]) + \mathrm{Var}(E[q_u \mid \hat{q}]) = (n_1\gamma/(n_1\gamma+1))^2(\sigma_q^2/n_1 + \sigma_p^2) = (n_1\gamma/(n_1\gamma+1))^2(\sigma_q^2(n_1\gamma+1)/n_1) = (n_1\gamma/(n_1\gamma+1))\sigma_p^2$. Finally, recall that we have normalized $q_p = 0$ (see §3). □

Proof of Proposition 1 (Outline). Using the expression for $\tau(p_1, p_2)$ in (2), it is straightforward to verify that for price plans satisfying $p_1 \le p_2$ profit is bounded by $\pi \le (1-c)^2/4$, and that the same is true for price plans satisfying both $p_1 > p_2$ and $p_1 - \delta_c p_2 > 1 - \delta_c$ simultaneously. Next, for price plans satisfying $p_1 > p_2$ and $p_1 - \delta_c p_2 \le 1 - \delta_c$, the firm's problem is concave and the optimal price plan is $\{p_1^*, p_2^*\}$, where $p_1^* = (c + c\delta_c + 2)/(\delta_c + 3)$ and $p_2^* = (2c + \delta_c + 1)/(\delta_c + 3)$ can be derived via first-order conditions; note also that $\pi(p_1^*, p_2^*) \ge (1-c)^2/4$ for all $\delta_c \in [0, 1]$, since $\pi(p_1^*, p_2^*) = (1-c)^2/(\delta_c + 3)$. The properties stated follow readily. □

Proof of Lemma 2. The proof proceeds in four steps. First, we show that any pure-strategy purchasing equilibrium must be characterized by a first-period threshold policy. Second, we establish the condition under which this threshold policy results in adoption inertia. Third, we show that when adoption inertia does not occur, the implicit equation that specifies the equilibrium threshold has exactly one solution, which lies in the interval $[p_1, 1]$. In the fourth step we establish the properties of this solution.

Step 1. Consider any arbitrary price plan $\{p_1, p_2\}$. Define customer i's best response function given that (any) ψ customers choose to purchase the product in the first period by $b_i(\psi)$ and note that $b_i(\psi) = 1$ (buy now) if $\Delta_i(\psi) = E[u_{i1}] - E[u_{i2}] \ge 0$, and $b_i(\psi) = 0$ (defer decision) if $\Delta_i(\psi) = E[u_{i1}] - E[u_{i2}] < 0$. We establish strict monotonicity of $\Delta_i(\psi)$ in x_i, thereby proving that the equilibrium must admit a first-period threshold structure. We have $\Delta_i(\psi) = x_i - p_1 - \delta_c E[(x_i + q_u - p_2)^+] = x_i - p_1 - \delta_c \int_{p_2 - x_i}^{+\infty} (x_i + q_u - p_2) f(q_u; 1-\psi)\, dq_u$, such that $q_u \sim \mathrm{N}(0, \psi\sigma_p^4/(\psi\sigma_p^2 + \sigma_q^2))$. The derivative with respect to x_i is given by $\partial\Delta_i/\partial x_i = 1 - \delta_c \int_{p_2 - x_i}^{+\infty} f(q_u; 1-\psi)\, dq_u = 1 - \delta_c \bar{F}(p_2 - x_i; 1-\psi) = 1 - \delta_c \bar{\Phi}((p_2 - x_i)/(\sigma(1-\psi))) > 0$. Since the above monotonicity holds for any arbitrary ψ customers, it follows that any pure-strategy equilibrium must admit a first-period threshold structure. Call this threshold $\theta(p_1, p_2)$.

Step 2. Given that any equilibrium must follow a threshold structure, we now identify the condition under which the first-period purchasing threshold is such that no customer purchases in the first period (i.e., $\theta(p_1, p_2) = 1$). To do so, we consider the highest x_i customer (i.e., $x_i = 1$). Her first-period expected utility from purchase is $1 - p_1$. If she delays, the threshold structure of the equilibrium implies that no customer buys in the first period, no reviews are generated, and her expected utility from purchasing in the second period is $\delta_c(1-p_2)$. Thus, the condition for adoption inertia is simply $1-p_1<\delta_c(1-p_2)$, or $p_1-\delta_c p_2>1-\delta_c$.

Step 3. If the above condition for adoption inertia does not hold, this implies that at least some customers will purchase the product in the first period. Denote customer i's expected utility from purchase in the first period by $v_1(x_i)$, $v_1(x_i)=x_i-p_1$, and customer i's expected utility from purchase in the second period, conditional on customers with $x_i\ge y$ choosing to purchase the product in the first period, by $v_2(x_i,y)$, $v_2(x_i,y)=\delta_c\int_{p_2-x_i}^{+\infty}(x_i+q_u-p_2)f(q_u;y)dq_u$. We will show that the indifference equation $v_1(y)=v_2(y,y)$ has exactly one solution, y^*, satisfying $y^*\in[p_1,1]$. Notice first that $\partial v_1/\partial y=1$. Next, rewrite $v_2(y,y)=\delta_c\int_{p_2-y}^{+\infty}(y+q_u-p_2)\cdot f(q_u;y)\,dq_u=\delta_c(y-p_2)\Phi((y-p_2)/\sigma(y))+\delta_c\sigma(y)\cdot\phi((y-p_2)/\sigma(y))=:u(y)$. The derivative of $u(y)$ yields $\partial u/\partial y=\delta_c(y-p_2)\cdot\phi((y-p_2)/\sigma(y))(\partial/\partial y)((y-p_2)/\sigma(y))+\delta_c\Phi((y-p_2)/\sigma(y))+\delta_c\sigma'(y)\phi((y-p_2)/\sigma(y))+\delta_c\sigma(y)\phi'((y-p_2)/\sigma(y))\cdot(\partial/\partial y)((y-p_2)/\sigma(y))=\delta_c(\Phi((y-p_2)/\sigma(y))+\sigma'(y)\phi((y-p_2)/\sigma(y)))$, and since $\sigma'(y)<0$, we have $\partial u/\partial y<1$. By comparing the derivatives of the two functions, it follows that $v_1(y)=v_2(y,y)$ can have at most one solution in the interval $y\in[0,1]$. From Step 2, we know that since adoption inertia does not occur, it must be the case that $v_1(1)\ge v_2(1,1)$. Moreover, note that $v_1(p_1)=0$ and $v_2(y,y)\ge 0$ for all $y\in[0,1]$ with equality only at $\delta_c=0$; therefore, we know that $v_1(p_1)\le v_2(p_1,p_1)$. Combining the above, we deduce that a unique solution y^* exists and satisfies $y^*\in[p_1,1]$.

Step 4. If $p_1-\delta_c p_2\le 1-\delta_c$, then we have $\theta\in[p_1,1]$. Furthermore, θ satisfies the equation $\theta-p_1-u(\theta)=0$ (†), where $u(\theta)=\delta_c\int_{p_2-\theta}^{+\infty}(\theta+q_u-p_2)f(q_u;\theta)\,dq_u$. The properties stated in the proposition can be obtained via the implicit function theorem as follows. Taking the total derivative of (†) with respect to p_1, we have $\partial\theta/\partial p_1-1-(\partial u/\partial\theta)(\partial\theta/\partial p_1)=(\partial\theta/\partial p_1)(1-\partial u/\partial\theta)-1=0$. From Step 3 we know that $\partial u/\partial\theta<1$, and therefore the above equation implies $\partial\theta/\partial p_1>0$. Next, the total derivative with respect to p_2 yields $\partial\theta/\partial p_2-(\partial u/\partial p_2+(\partial u/\partial\theta)(\partial\theta/\partial p_2))=\partial\theta/\partial p_2-(-\delta_c\bar{F}(p_2-\theta;\theta)+(\partial u/\partial\theta)(\partial\theta/\partial p_2))=(\partial\theta/\partial p_2)(1-\partial u/\partial\theta)+\delta_c\bar{F}(p_2-\theta;\theta)=0$. Therefore, $\partial\theta/\partial p_2<0$, which implies that the number of first-period buyers is increasing in p_2. As for the result pertaining to customers' degree of patience δ_c, we have $\partial\theta/\partial\delta_c-(\partial u/\partial\delta_c+(\partial u/\partial\theta)(\partial\theta/\partial\delta_c))=(\partial\theta/\partial\delta_c)(1-\partial u/\partial\theta)-\partial u/\partial\delta_c=0$. Since $\partial u/\partial\delta_c>0$, it follows that $\partial\theta/\partial\delta_c>0$. Finally, note that we may rewrite $u(\theta)=\delta_c(\theta-p_2)\Phi((\theta-p_2)/\sigma(\theta))+\delta_c\sigma(\theta)\phi((\theta-p_2)/\sigma(\theta))$, from which it follows that $\partial u/\partial\sigma_p=\delta_c(\partial\sigma(\theta)/\partial\sigma_p)\phi((\theta-p_2)/\sigma(\theta))>0$, since $\sigma(\theta)=\sigma_p\sqrt{(1-\theta)\gamma/((1-\theta)\gamma+1)}$. Taking the total derivative of (†) with respect to σ_p, $\partial\theta/\partial\sigma_p-(\partial u/\partial\sigma_p+(\partial u/\partial\theta)(\partial\theta/\partial\sigma_p))=(\partial\theta/\partial\sigma_p)(1-\partial u/\partial\theta)-\delta_c(\partial\sigma(\theta)/\partial\sigma_p)\cdot\phi((\theta-p_2)/\sigma(\theta))=0$. Therefore, $\partial\theta/\partial\sigma_p>0$, which implies that the number of first-period buyers is decreasing in σ_p. In a similar manner, it can also be verified that $\partial\theta/\partial\sigma_q<0$, and since $\gamma=\sigma_p^2/\sigma_q^2$, the two latter results imply that $\partial\theta/\partial\gamma>0$. In the second period, consumers remaining in the market satisfy $x_i<\theta(p_1,p_2)$ and purchase only if $x_i\ge p_2-q_u$. □

Proof of Proposition 2. Uniqueness of the equilibrium in the pricing-adoption game for any price plan $\{p_1,p_2\}$ follows readily from uniqueness of the purchasing equilibrium in Lemma 2. We first prove that adoption inertia can never be optimal for the firm. Note that among all price plans that induce adoption inertia, those that achieve the highest profit have second-period price $p_2=(1+c)/2$ and achieve total profit of $\pi_{in}^*=(1-c)^2/4$. To prove that adoption inertia cannot be optimal, we will show that an alternative price plan (which does not induce adoption inertia) achieves profit strictly higher than $(1-c)^2/4$. Since this price plan is not necessarily optimal for the firm, it follows that adoption inertia cannot be optimal. Consider the price plan $\{(1+c)/2,(1+c)/2\}$. In this case, first-period profit is $\pi_1=((1-c)/2)(1-\theta)$ for some $\theta\in[(1+c)/2,1]$ (see Lemma 2). Denoting second-period expected profit by π_2, we will show that $\pi_1+\pi_2>(1-c)^2/4$. Equivalently, we will show $\pi_2=((1-c)/2)E[s_2]>(1-c)^2/4-\pi_1=((1-c)/2)((1-c)/2-1+\theta)$, where s_2 denotes second-period sales—this reduces to showing $E[s_2]>\theta-(1+c)/2$. Note that $q_u\sim\mathrm{N}(0,\sigma(\theta))$ and that (i) $s_2(q_u)=0$ if $q_u\le(1+c)/2-\theta$ (ii) $s_2(q_u)=\theta+q_u-(1+c)/2$ if $(1+c)/2-\theta<q_u\le(1+c)/2$, and (iii) $s_2(q_u)=\theta$ if $q_u>(1+c)/2$. Next, decompose $s_2(q_u)$ into $s_2(q_u)=s_a(q_u)+s_b(q_u)$, where

$$s_a(q_u)=\begin{cases}0 & \text{if } q_u\le\dfrac{1+c}{2}-\theta,\\[2mm] \theta+q_u-\dfrac{1+c}{2} & \text{if } \dfrac{1+c}{2}-\theta<q_u\le\theta-\dfrac{1+c}{2},\\[2mm] 2\theta-(1+c) & \text{if } q_u>\theta-\dfrac{1+c}{2};\end{cases}$$

$$s_b(q_u)=\begin{cases}0 & \text{if } q_u\le\theta-\dfrac{1+c}{2},\\[2mm] -\theta+q_u+\dfrac{1+c}{2} & \text{if } \theta-\dfrac{1+c}{2}<q_u\le\dfrac{1+c}{2},\\[2mm] -\theta+(1+c) & \text{if } q_u>\dfrac{1+c}{2}.\end{cases}$$

We have $E[s_2]=\int_{-\infty}^{+\infty}s_2(q_u)f(q_u;\theta)\,dq_u=\int_{-\infty}^{+\infty}(s_a(q_u)+s_b(q_u))\cdot f(q_u;\theta)\,dq_u$. The symmetry of the r.v. q_u around zero and the function $s_a(q_u)$ imply $\int_{-\infty}^{+\infty}s_a(q_u)f(q_u;\theta)\,dq_u=\theta-(1+c)/2$. Moreover, since $s_b(q_u)$ is a nonnegative function of q_u (and positive for some values of q_u), it follows that $\int_{-\infty}^{+\infty}s_b(q_u)f(q_u;\theta)\,dq_u>0$. Therefore, we have $E[s_2]>\theta-(1+c)/2$, and the first point of the proposition is proven. For the second point of the proposition, using the above, we may restrict our attention to policies $\{p_1,p_2\}$ which result in $\theta\in[p_1,1]$. The indifference equation that connects the price plan to the first-period purchasing threshold, θ, is $\theta-p_1=\delta_c\int_{p_2-\theta}^{+\infty}(\theta+q_u-p_2)f(q_u;\theta)\,dq_u$ ($\star$). We make use of the following lemma.

Lemma 5. *For any given preannounced price plan $\{p_1,p_2\}$, an individual customer's second-period expected utility is strictly increasing in the number of first-period buyers.*

Proof. Let $v_2(x_i,y):=\delta_c\int_{p_2-x_i}^{+\infty}(x_i+q_u-p_2)f(q_u;y)dq_u$ denote the second-period expected utility of customer x_i, conditional on $n_1=1-y$ customers purchasing in the first period. We will show that $\partial v_2/\partial y<0$. Rewrite

$$v_2(x_i,y)=\delta_c\int_{p_2-x_i}^{+\infty}(x_i+q_u-p_2)f(q_u;y)\,dq_u$$
$$=\delta_c\int_{-\infty}^{+\infty}m_i(q_u)f(q_u;y)\,dq_u,$$

where $m_i(q_u)=0$ if $q_u<p_2-x_i$, and $m_i(q_u)=x_i+q_u-p_2$ otherwise. Note that $m_i(q_u)$ is nonnegative, convex, and increasing

in q_u. The preposterior distribution of q_u is Normal, which satisfies the convex order (see Müller and Stoyan 2002, p. 63). It therefore follows that the integral $\int_{-\infty}^{+\infty} m_i(q_u) f(q_u; y)\, dq_u$ is strictly increasing in the preposterior variance. In turn, this variance is decreasing in y. Thus, the proof of the lemma is complete.

From Lemma 5, we know that since a positive number of customers purchase in the first period, for any $\gamma > 0$ we have $\delta_c \int_{p_2-\theta}^{+\infty} (\theta + q_u - p_2) f(q_u; \theta)\, dq_u > \delta_c(\theta - p_2)$; that is, the marginal customer's second-period expected utility in the presence of SL ($\gamma > 0$) is greater than that in its absence ($\gamma \to 0$). Therefore, ($\star$) implies that the equilibrium price plan and θ satisfy $\theta - p_1 > \delta_c(\theta - p_2)$. At the extreme case of $\delta_c = 1$, the last inequality implies $p_2^* > p_1^*$. To complete the proof, note that as a consequence of the maximum theorem (see Sundaram 1996, Theorem 9.14), the optimal price plan $\{p_1^*, p_2^*\}$ is upper-semicontinuous in $\delta_c \in [0, 1]$. This implies the existence of a threshold $\Delta(\gamma) \in [0, 1]$ such that $p_2^* > p_1^*$ for $\delta_c \geq \Delta(\gamma)$. $\square$

Proof of Example 1. For $\delta_c = 0$, we have $\theta(p_1, p_2) = p_1$ for all $\{p_1, p_2\}$. Consider some arbitrary $p_1 > 0$; denoting by p_2^* the optimal second-period price we want to show that for any $\gamma > 0$ we have $p_2^* > (p_1 + c)/2$ (note that for $\gamma \to 0$ we have $p_2^* = (p_1 + c)/2$). The total expected profit as a function of the decision p_2 is $\pi(p_2) = (p_1 - c)(1 - p_1) + (p_2 - c)\int_{p_2-p_1}^{p_2} (p_1 + q - p_2) f(q; p_1)\, dq + (p_2 - c)\int_{p_2}^{+\infty} p_1 f(q; p_1)\, dq$. To prove that $p_2^* > (p_1 + c)/2$, it suffices to show that the profit function is strictly increasing in p_2 in the interval $p_2 \in [0, (p_1 + c)/2]$. We have $d\pi/dp_2 = 2\int_{p_2-p_1}^{p_2} ((p_1 + c)/2 - p_2 + q/2) f(q; p_1)\, dq + \int_{p_2}^{+\infty} p_1 f(q; p_1)\, dq$, which is strictly positive for $0 \leq p_2 \leq (p_1 + c)/2$. $\square$

Proof of Proposition 3. For the extreme case of $\delta_c = 0$, Example 2 (see proof below) establishes that the firm's profit is greater in the presence of SL. Consider next the other extreme case of $\delta_c = 1$. Denote firm profit at prices $\{p_1, p_2\}$ in the absence of SL by $\pi_n(p_1, p_2)$. From Proposition 1 it follows that when $\delta_c = 1$, $\pi_n^* = \max_{p_1, p_2} \pi_n = (1 - c)^2/4$. In the presence of SL, the firm can achieve profit equal to π_n^* by inducing adoption inertia (see the first part of the proof of Proposition 2). But from Proposition 2, we know that adoption inertia is strictly suboptimal for the firm for any $\gamma > 0$. Thus, we have established that $\pi^*|_{\gamma>0} > \pi_n$ when $\delta_c = 1$. Next, note the firm's profit is continuous in $\delta_c \in [0, 1]$, and, as a direct implication of the maximum theorem (Sundaram 1996, see Theorem 9.14), the firm's optimal profit function is also continuous in $\delta_c \in [0, 1]$. Thus, for any $\gamma > 0$ there exist thresholds such as those stated in the proposition.

Consider the case of fixed pricing. In the presence of SL, from the first part of the proof of Proposition 2 we know that if the firm chooses the price plan $p_1 = p_2 = (1 + c)/2$ it achieves expected profit strictly higher than $(1 - c)^2/4$, which is the optimal profit in the absence of SL; thus, for $\delta_c = 1$, firm profit is higher in the presence of SL. For the case of $\delta_c = 0$, in the absence of SL the firm's optimal profit is also $(1 - c)^2/4$, and the optimal price is $(1 + c)/2$. If the firm charges $p_1 = p_2 = (1 + c)/2$, its expected profit is strictly higher in the presence of SL (apart from the case of $c = 0$, where its expected profit is equal to that under no SL). Thus, the result of the proposition continues to hold even when the firm employs fixed pricing. $\square$

Proof of Example 2. Let $\delta_c = 0$, let the optimal price plan when $\gamma \to 0$ be $\{p_1^n, p_2^n\}$, and note that $p_2^n = (p_1^n + c)/2$. For any $\gamma = k > 0$, fix $p_1 = p_1^n$. Since we have fixed p_1 and customers are myopic, the firm's first-period profit is identical for the two cases $\gamma \to 0$ and $\gamma = k$, and we need only consider differences in expected second-period profit. Assume that the firm announces $\hat{p}_2 = (p_1^n + c)/2$ irrespective of γ. Denoting second-period profit at second-period price p_2 by $\pi_2(p_2)$, we will show that $E[\pi_2(\hat{p}_2)]|_{\gamma=k} \geq E[\pi_2(\hat{p}_2)]|_{\gamma\to 0}$. Since $\hat{p}_2$ is optimal for $\gamma \to 0$ but suboptimal for $\gamma = k$ (as illustrated in Example 1), this implies that the firm achieves strictly higher expected second-period profit, and therefore strictly higher overall expected profit, when $\gamma = k$. The firm's second-period profit at $\hat{p}_2 = (p_1^n + c)/2$ is (i) $\pi_2(\hat{p}_2) = 0$ if $q_u \leq (c - p_1^n)/2$, (ii) $\pi_2(\hat{p}_2) = ((p_1^n - c)/2)(q + (p_1^n - c)/2)$ if $(c - p_1^n)/2 < q_u \leq (p_1^n + c)/2$, and (iii) $\pi_2(\hat{p}_2) = ((p_1^n - c)/2)p_1^n$ if $q_u > (p_1^n + c)/2$. Next, note that $E[\pi_2(\hat{p}_2)]|_{\gamma\to 0} = (p_1^n - c)^2/4$. Using a simple decomposition argument for $\pi_2(\hat{p}_2)$ (analogous to the one used for $s_2(q_u)$ in the proof of Proposition 2), it is straightforward to show $E[\pi_2(\hat{p}_2)]|_{\gamma=k} - E[\pi_2(\hat{p}_2)]|_{\gamma\to 0} \geq 0$ (equality holds only when $c = 0$). We conclude that $\pi^*|_{\gamma=k} > \pi^*|_{\gamma\to 0}$ for any $k > 0$. $\square$

Proof of Proposition 4 (Outline). For $p_1 > (2 - \delta_c \cdot (1 - c))/2$ we have $\pi(p_1) = (1 - c)^2/4$, because no sales occur in the first period, and the optimal second-period price is $(1 + c)/2$. Next, for $p_1 \leq (2 - \delta_c(1 - c))/2$ we have $\pi(p_1) = (p_1 - c)(1 - \chi(p_1)) + (\chi(p_1) - c)^2/4$. This profit function is strictly concave in p_1 with a unique maximizer at $p_1^* = (2c + \delta_c^2(1 - c) + 4(1 - \delta_c))/(6 - 4\delta_c)$. Note that $p_1^* \leq (2 - \delta_c(1 - c))/2$ (i.e., $p_1^* - (2 - \delta_c(1 - c))/2 = (1 - c)(\delta_c^2 - 3\delta_c + 2)/(2(2\delta_c - 3)) \leq 0$ for $\delta_c \in [0, 1]$), and the profit function is continuous; this implies that the global maximizer of the profit function is p_1^*. The second-period price follows from the discussion of the second-period subgame, whereas the properties stated at the end of the proposition follow readily. $\square$

Proof of Lemma 3. In the second period, the firm faces a mass of $\bar{x}$ customers with valuations uniformly distributed $U[q_u, q_u + \bar{x}]$. If $q_u \leq c - \bar{x}$, then no customer will purchase the product in the second period at any profitable price $p_2 > c$. In this case, the firm exits the market; this is denoted by an optimal price $p_2^* = c$, and second-period profit is $\pi_2 = 0$. If $q_u > c - \bar{x}$, then the firm's profit function is $\pi_2(p_2) = (p_2 - c)\min\{q_u + \bar{x} - p_2, \bar{x}\}$. Any price $p_2 < q_u$ cannot be optimal. We therefore restrict our attention to the function $\pi_2(p_2) = (p_2 - c)(q_u + \bar{x} - p_2)$. If $q_u > c + \bar{x}$, the profit function is decreasing for $p_2 \geq q_u$, and we have $p_2^* = q_u$ with associated profits $\pi_2^* = (q_u - c)\bar{x}$. If $c - \bar{x} < q_u \leq c + \bar{x}$, the profit function is increasing at $p_2 = q_u$ and concave, and we have $p_2^* = (q_u + c + \bar{x})/2$ with associated profits $\pi_2^* = ((q_u + \bar{x} - c)/2)^2$. The customer's second-period purchasing decision is trivial: she purchases provided $x_i + q_u - p_2^*$ is nonnegative. $\square$

Proof of Lemma 4. The proof proceeds in four steps. First, we show that any pure-strategy purchasing equilibrium must be characterized by a first-period threshold policy. Second, we establish the condition under which adoption inertia occurs. Third, we show that when adoption inertia does not occur, the implicit equation that specifies the equilibrium threshold has exactly one solution, which lies in the interval $[p_1, 1]$. In the fourth step we establish the properties of this solution.

Step 1. By contradiction. Suppose that for some p_1 there exists a pure-strategy purchasing equilibrium where a total of ψ customers purchase in the first period, and let the firm's second-period price plan be $p_2^*(q_u)$. Suppose that the purchasing equilibrium is not of a threshold type; i.e., there exists some customer with $x_i = x_l$ who purchases in the first period, whereas another customer with $x_i = x_h > x_l$ does not. Let $\Delta_i(\psi) = x_i - p_1 - \delta_c E[(x_i + q_u - p_2^*(q_u))^+]$. In such an equilibrium, we have $\Delta_l(\psi) \geq 0$, $\Delta_h(\psi) < 0$ so that $\Delta_h(\psi) - \Delta_l(\psi) < 0$. Define the sets $Q^l = \{q_u\colon x_l + q_u - p_2^*(q_u) \geq 0\}$ and $Q^h = \{q_u\colon x_h + q_u - p_2^*(q_u) \geq 0\}$, and note that $Q^l \subseteq Q^h$. Define also $Q^c = \{q_U\colon q_u \in Q^h, q_u \notin Q^l\}$ and note that if $q_u \in Q^c$, then $x_h + q_u - p_2^*(q_u) < x_h - x_l$. We have $\Delta_h(\psi) - \Delta_l(\psi) = (x_h - x_l) - \delta_c(\int_{q\in Q^l}(x_h + q_u - p_2^*(q_u))\, dF(q_u; 1-\psi) + \int_{q\in Q^c}(x_h + q_u - p_2^*(q_u))\, dF(q_u; 1-\psi) - \int_{q\in Q^l}(x_l + q_u - p_2^*(q_u))\, dF(q_u; 1-\psi)) > (x_h - x_l) - \delta_c(\int_{q\in Q^l}(x_h - x_l)\, dF(q_u; 1-\psi) + \int_{q\in Q^c}(x_h - x_l)\, dF(q_u; 1-\psi)) > 0$, which leads to a contradiction.

Step 2. Adoption inertia occurs if the highest first-period valuation consumer (i.e., a consumer with $x_i = 1$) prefers not to purchase in the first period. This happens when $1 - p_1 < \delta_c((1-c)/2)$, or, equivalently, when $p_1 > (2 - \delta_c(1-c))/2$.

Step 3. The first-period purchasing threshold at first-period price p_1 solves the indifference equation

$$x - p_1 = \delta_c\left(\int_{c-x}^{c+x}\left(x + q_u - \frac{q_u + c + x}{2}\right) f(q_u; x)\, dq_u + \int_{c+x}^{+\infty} x f(q_u; x)\, dq_u\right), \tag{9}$$

where the LHS (RHS) of the equation denotes the marginal customer's first-period (second-period) expected utility. The LHS has derivative w.r.t. x equal to one. We will show that a solution to the above equation, if it exists, is unique by showing that the derivative of the RHS is strictly less than one. Define $u(x) := \int_{c-x}^{c+x}(x + q_u - (q_u + c + x)/2) f(q_u; x)\, dq_u + \int_{c+x}^{+\infty} x f(q_u; x)\, dq_u = ((x-c)/2) \cdot (\Phi((x-c)/\sigma(x)) - \Phi((-x-c)/\sigma(x))) + \frac{1}{2}\sigma(x)(\phi((c-x)/\sigma(x)) - \phi((c+x)/\sigma(x))) + x\Phi((-x-c)/\sigma(x))$. Differentiating w.r.t. x (and after some manipulation), $u'(x) = \frac{1}{2}(\Phi((x-c)/\sigma(x)) - \Phi((-x-c)/\sigma(x))) + \frac{1}{2}\sigma'(x)(\phi((c-x)/\sigma(x)) - \phi((c+x)/\sigma(x)))$. In the last expression, note that (i) $0 \leq \Phi((x-c)/\sigma(x)) - \Phi((-x-c)/\sigma(x)) \leq 1$, (ii) $\sigma'(x) < 0$ (variance of preposterior decreases in x), and (iii) $\phi((c-x)/\sigma(x)) - \phi((c+x)/\sigma(x)) \geq 0$ (by symmetry of the standard Normal density ϕ). Therefore, we have $u'(x) < \frac{1}{2} < 1$ so that a solution to (9), if it exists, is unique. Finally, note that (i) for $x = p_1$, $0 = LHS \leq RHS$ in (9) (equality holds when $\delta_c = 0$), and (ii) if $p_1 \leq (2 - \delta_c(1-c))/2$ (i.e., no adoption inertia), we have $LHS < RHS$. Thus, if $p_1 \leq (2 - \delta_c(1-c))/2$, a solution to (9) exists and is unique. (Note also that $u(0) = 0$ and that $u'(x) < \frac{1}{2}$. This implies that any solution to the indifference equation must satisfy $x < 2p_1$.)

Step 4. For the case of $p_1 \leq (2 - \delta_c(1-c))/2$, we next show that the solution to the indifference Equation (9) is (i) decreasing in c and (ii) increasing in γ. For $u(x)$ defined as above, the indifference equation is $x - p_1 - u(x) = 0$ ($\star$). To prove (i), take the total derivative of ($\star$) with respect to c, $\partial x/\partial c - (\partial u/\partial c + (\partial u/\partial x)(\partial x/\partial c)) = 0$, $(\partial x/\partial c)(1 - \partial u/\partial x) = \partial u/\partial c$. From the preceding analysis we have $\partial u/\partial x < 1$, which implies that the sign of $\partial x/\partial c$ is the same as the sign of $\partial u/\partial c$. Thus, $\partial u/\partial c = \int_{c-x}^{c+x} -\frac{1}{2} f(q_u; x)\, dq_u + x f(c+x; x) - x f(c+x; x) = -\frac{1}{2}\int_{c-x}^{c+x} f(q_u; x)\, dq_u < 0$, and therefore $\partial x/\partial c < 0$. To prove (ii), take the total derivative of ($\star$) with respect to γ to get $(\partial x/\partial\gamma)(1 - \partial u/\partial x) = \partial u/\partial\gamma$. Thus, $\partial x/\partial\gamma$ has the same sign as $\partial u/\partial\gamma$. Note that $\partial u/\partial\gamma = (\partial u/\partial\sigma(x))(\partial\sigma(x)/\partial\gamma)$ and $\partial\sigma(x)/\partial\gamma > 0$. Next, write

$$u(x) = \frac{x-c}{2}\left(\Phi\left(\frac{x-c}{\sigma(x)}\right) - \Phi\left(\frac{-x-c}{\sigma(x)}\right)\right) + \frac{1}{2}\sigma(x)\left(\phi\left(\frac{x-c}{\sigma(x)}\right) - \phi\left(\frac{-x-c}{\sigma(x)}\right)\right) + x\Phi\left(\frac{-x-c}{\sigma(x)}\right),$$

and $\partial u/\partial\sigma(x) = \frac{1}{2}[\phi((x-c)/\sigma(x)) - \phi((-x-c)/\sigma(x))]$. Note that if $c = 0$, we have $\partial u/\partial\sigma(x) = 0$ so that $\partial x/\partial\gamma = 0$. However, if $c > 0$, we have $\partial u/\partial\sigma(x) > 0$, which implies $\partial x/\partial\gamma > 0$. It is straightforward to deduce in similar fashion that the solution to (9) is also increasing in δ_c and in p_1. □

Proof of Proposition 5. Uniqueness of the equilibrium in the pricing-adoption game follows from Lemmas 4 and 3. Furthermore, given the firm's chosen first-period price p_1^*, the second point of the proposition (which defines the second-period price) follows directly by substituting p_1^* and the purchasing threshold $\zeta(p_1^*)$ into the result of Lemma 3. It remains to prove the first point of the proposition, which states that adoption inertia is strictly suboptimal for the firm. When $p_1 > 1 - \delta_c(1-c)/2$ (i.e., in the case of adoption inertia), the second-period optimal price is $p_2 = (1+c)/2$, and $1 - (1+c)/2$ consumers purchase, so that firm profit is $\pi = (1-c)^2/4$. We will show that there exists an alternative pricing policy, which does not induce adoption inertia, that performs better. Suppose the firm announces a first-period price $p_1 = (1+c)/2$, and note that since $c \in [0, 1)$ we have $(1+c)/2 \leq 1 - \delta_c(1-c)/2$, which guarantees that adoption inertia does not occur. According to Lemma 4, at price $p_1 = (1+c)/2$ the first-period purchasing threshold is some $\zeta \in [p_1, 1]$. Now suppose that in the second period and irrespective of the realization of q_u, the firm maintains the same price, i.e., $p_2 = (1+c)/2$. Importantly, note that both the first-period price we assume here as well as the second-period price (given p_1, ζ, and the realization q_u) are generally suboptimal; nevertheless, we show that this suboptimal policy performs better than adoption inertia. Since sales under these two pricing regimes occur at the same price, it will suffice to show that the total number of expected sales under the second regime is higher than that under inertia. Under inertia, we have $s_n = (1 - (1+c)/2) = (1-c)/2$, where s_n denotes total sales. Under the alternative policy, the total sales s are given by

$$s = \begin{cases} 1 - \zeta & \text{if } q_u \leq \dfrac{1+c}{2} - \zeta, \\[2ex] 1 + q_u - \dfrac{1+c}{2} & \text{if } \dfrac{1+c}{2} - \zeta < q_u \leq \dfrac{1+c}{2}, \\[2ex] 1 & \text{if } q_u > \dfrac{1+c}{2}. \end{cases}$$

Since q_u is a zero-mean Normal random variable, a straightforward decomposition of s above reveals that $E[s] \geq s_n$, where equality holds only for $c = 0$. The result of the proposition follows. □

Proof of Example 3. Fix p_1 at the optimal first-period price when $\delta_c = 0$ and SL is absent (i.e., $\gamma \to 0$). Consider what happens as γ increases from zero: since the firm's second-period profit is nonnegative, increasing, and convex in q_u, it follows that the firm's expected second-period profit is increasing in the variance of the preposterior distribution of q_u, which in turn is increasing in γ. Moreover, note that the first-period price used in the above argument is not necessarily optimal for $\gamma = k > 0$, which implies that the firm's profit is strictly higher for $\gamma = k$ than it is for $\gamma \to 0$. □

Proof of Proposition 6. In the extreme case $\delta_c = 0$, Example 3 suggests that firm profit is strictly higher in the presence of SL. Next, consider case $\delta_c = 1$. In the absence of SL, the best the firm can achieve is the optimal single-period profit $\pi^* = ((1-c)/2)^2$. In the presence of SL, this profit can be achieved by the firm by inducing adoption inertia. However, from Proposition 5, we know that adoption inertia is suboptimal for the firm in the presence of SL; therefore, the firm's optimal profit in the presence of SL is greater than that in its absence. Next, note the that firm's profit is continuous in $\delta_c \in [0, 1]$ and, as a direct implication of the maximum theorem (see Sundaram 1996, Theorem 9.14), the firm's optimal profit function is also continuous in $\delta_c \in [0, 1]$—this implies existence of the thresholds stated in the proposition. □

Proof of Proposition 7. Using the optimal prices stated in Propositions 1 and 4, we may calculate the difference in optimal firm profit under preannounced and responsive pricing as $\Delta\pi_b^* = \pi_{bp}^* - \pi_{br}^* = \delta_c^2(1-c)^2(1-\delta_c)/(4(9-2\delta_c^2-3\delta_c))$. Thus, we have $\Delta\pi_b^* \geq 0$ for any $\delta_c \in [0, 1]$, where the inequality is strict for $\delta_c \in (0, 1)$. □

Proof of Proposition 8. Let $\gamma = k > 0$ and consider the extreme $\delta_c = 0$. Let $\{p_1^*, p_2^*\}$ be the optimal preannounced price plan. Consider a responsive price plan $\{p_1, p_2\}$ with $p_1 = p_1^*$, $p_2 = p_2^*$ for $q_u \geq p_2^* - p_1^*$, and $p_2 = p_2^* - \epsilon$ for $q_u < p_2^* - p_1^*$, where ϵ is small and positive (this price plan is not necessarily optimal). These two pricing policies achieve identical first-period profit, identical reviews, and identical second-period profit for any realization of q_u such that $q_u \geq p_2^* - p_1^*$. However, the responsive price plan achieves higher second-period profit for at least some $q_u < p_2^* - p_1^*$: under all such scenarios, the preannounced price plan achieves zero second-period profit, whereas the responsive price plan achieves positive profit under at least some scenarios. Since scenarios under which the responsive price plan achieves higher second-period profit occur with positive probability, it follows that this suboptimal responsive policy outperforms the optimal preannounced policy. Thus we have $\pi_r^*|_{\gamma=k,\,\delta_c=0} - \pi_p^*|_{\gamma=k,\,\delta_c=0} > 0$. Next, note that π_r^* and π_p^* are both continuous in $\delta_c \in [0, 1]$—this is a direct implication of the continuity of both profit functions in $\delta_c \in [0, 1]$ and the maximum theorem (see Sundaram 1996, Theorem 9.14)—which implies existence of the threshold described in the proposition. □

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