

Scarcity Strategies Under Quasi-Bayesian Social Learning

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This paper proposes a theory of firm-induced scarcity in the early stages of a new product's selling season. By restricting supply, the firm induces self-selection of customers who are likely to generate the most positive product reviews; this can be advantageous for the firm if, as suggested by existing empirical evidence, subsequent consumers do not appropriately account for the selection bias in the early review sample. We develop a novel model of *quasi-Bayesian* social learning that allows us to test the implications of this theory. Our analysis yields three main insights. First, the described rationale constitutes a profitable strategy for the firm only in settings characterized by price rigidity and only if the product's quality is not too high with respect to prior market expectations; for such cases, we provide guidelines on optimal pricing and quantity decisions. Second, although induced scarcity represents an effort by the firm to manipulate the social learning process, this may result not only in an increase in the firm's profit, but also in the consumers' surplus. Third, product scarcity may act as an effective substitute for dynamic pricing, allowing the firm to approximate dynamic-pricing outcomes while charging a fixed price.

1. Introduction

Apple products always seem to be stocked out for several weeks following their introduction. Having developed a culture of early supply shortages, the computing giant induces eager consumers to compete with one another to secure a unit before the product stocks-out. The result is the recurring theme of queues building up, with thousands camping outside Apple stores all over the world for days in advance of product launch (e.g., *The Los Angeles Times* 2011). The official story behind these shortages is typically one of unexpectedly high demand coupled with constraints on production capacity (e.g., *Reuters* 2012). However, the frequency of such stock-outs often raises suspicions that this may in fact be a deliberate strategy; and these suspicions appear to be consistent with the words of a former high-ranking employee:

“When we were planning the launch of the iPod across Europe, one of the important things we had to manage was to make sure we under-supplied the demand, so that we'd only roll it out almost in response to cities crying out for those iPods to become available – that's how we kept that kind of ‘cachet’ for the iPod in its early years. And we'd use extensive data research to understand what the relative strength of doing that in Rome versus Madrid would be.”

Andrew McGuinness, former European head of Apple's advertising agency (BBC 2011).

Apart from the notable example of Apple, early stock outs are commonly observed for other experiential products including high-tech consumer electronics such as Samsung’s Galaxy (*The Guardian* 2013) and Palm’s Pre (*CNET* 2009), and gaming consoles such as Sony’s PlayStation (*The Wall Street Journal* 2014) and Microsoft’s Xbox (*CNN* 2013). Most of the aforementioned firms have been accused, from time to time, of “shortage conspiracies” (e.g., *Bloomberg Businessweek* 2005).

In the existing academic literature, there are two main theories as to why a firm would deliberately induce product scarcity. The first is that of “buying frenzies,” which shows that early supply shortages can be beneficial either when prices are dynamic or product scarcity persists in the long-term (DeGraba 1995); yet scarcity strategies appear to be employed even when prices are fixed and availability in the long-term is ample. The second is a signalling-based theory, which shows that scarcity can be beneficial provided that consumers who are uninformed about a new product’s quality cannot observe the purchasing decisions of (or communicate with) other, better-informed consumers (Stock and Balachander 2005). However, stock-out events in the examples mentioned above tend to be widely-publicized in popular media, and consumers often discuss their purchasing and consumption experiences in online platforms. In this paper, we propose and investigate an alternative explanation for firm-induced scarcity, which may apply to settings where the existing theories do not.

Our explanation is based on social learning (SL), a term we use throughout to describe the process by which potential buyers learn from the reviews of consumers who have already purchased the product. The theory is simple and consists of the following two components:

- (1) Product scarcity induces competition among customers which, with the aid of an appropriate rationing mechanism, results in the limited supply being allocated to customers who are *ex ante* likely to generate the most positive product reviews.
- (2) This biased sample of reviews influences customers remaining in the market, resulting in increased overall product adoption.

To illustrate the first point, we present the anecdotal observations of Figure 1, where early stock outs of Apple products were accompanied by higher average ratings (as compared to the products’ long-run average ratings). We believe these observations to be associated with the allocation of the limited supply on the basis of “waiting lines” (i.e., queues), which are known to result in *efficient* rationing of demand (Holt and Sherman 1982): put simply, consumers who are *ex ante* most likely to enjoy their experience with the new product (and therefore to generate a positive review *ex post*) are also those who are most likely to queue up first (and therefore to be among those who receive a unit before the product stocks out). The second point is the essence of a SL phenomenon that has been empirically-documented by Li and Hitt (2008): when engaging in SL, owing to cognitive and/or informational limitations, customers are unwilling and/or unable to account appropriately

for selection bias in product reviews. Our theory is that the two points put together may create a rationale for the firm to deliberately induce scarcity in the early stages of the selling season.

To investigate this theory and its implications, we consider a stylized model where a monopolist firm sells a new experiential product to a fixed population of consumers with heterogeneous preferences. The product is sold over two periods, with the first period representing the product's launch phase. In the event that demand in the launch phase outstrips supply, excess demand is rationed efficiently. Any consumer who purchases the product in the first period posts a review which reflects her ex post opinion of product quality: this opinion depends on the product's inherent quality (which is ex ante unknown and valued equally by all consumers) but is also influenced by the buyer's idiosyncratic preferences. Consumers remaining in the market in the second period observe the first-period reviews before deciding whether to purchase in the second period.

To capture the empirical finding of Li and Hitt (2008), which is at the core of our scarcity theory, we develop a *quasi-Bayesian* model of SL (see Camerer et al. 2003, Rabin 2013). Our model is parameterized so as to nest fully-rational learning (i.e., Bayes' rule) as a special case, but our focus is on the case where learning is less-than-fully rational and consumers do not (sufficiently) account for the selection bias in the launch-phase reviews. The firm seeks to maximize its profit by controlling the product's price and the quantity made available in each selling period. A scarcity strategy is defined as one where the firm makes fewer units available in the first period than the number of units demanded at the firm's chosen price.

We consider first the case of fixed pricing, consistent with our motivating examples, and seek to verify the profitability of a scarcity strategy.¹ Restricting supply in the launch phase allows the firm to achieve a higher average rating (as in Figure 1), however, it also entails a smaller volume of launch-phase reviews. The latter has the effect of product reviews carrying less "weight" in influencing second-period consumers; as a result, the decision to induce scarcity is not straightforward. We show that the firm's pricing-and-quantity problem has a unique solution, and derive a necessary and sufficient condition under which a scarcity strategy is optimal. We demonstrate that this condition can be expressed as an upper threshold on the product's quality, whereby scarcity is beneficial provided the product's quality is not significantly higher than customers' ex ante expectations – in these cases, we provide guidelines on how optimal pricing-and-quantity strategies, as well as firm profit, depend on the product's and the consumers' characteristics.

We next consider the welfare implications of scarcity strategies. We observe that when the firm optimizes both the product's price and the launch-phase quantity, this generally leads to an increase in total welfare. Interestingly, in many cases where the firm induces deliberate stock outs, we find

¹ On one occasion, Apple reduced the iPhone's price just two months after its launch, a decision that resulted in significant customer backlash; the firm was eventually forced to refund early buyers (*The New York Times* 2007).

that the increase in total welfare is attributed not only to an increase in the firm’s profit, but also to an increase in consumer surplus. In particular, the net effect of product scarcity on consumer surplus depends on a comparison between two opposing effects: on one hand, biased reviews under product scarcity cause some customers to purchase the product erroneously, leading to losses in consumer surplus; on the other, product scarcity is typically accompanied by the firm with lower prices, resulting in a surplus gain.

Finally, we consider the case where the firm employs dynamic pricing. We demonstrate that (in the context of our theory) product scarcity cannot be beneficial for the firm under dynamic pricing, and highlight that scarcity strategies under fixed pricing effectively act as a substitute for dynamic pricing. Pursuing this line of thought, we find that product scarcity may allow the firm to achieve profits close to those under dynamic pricing while charging a fixed price. This finding may be of particularly importance in settings characterized by price-rigidity, where firms are reluctant or unable to price dynamically, for example, because of fairness considerations or market norms.

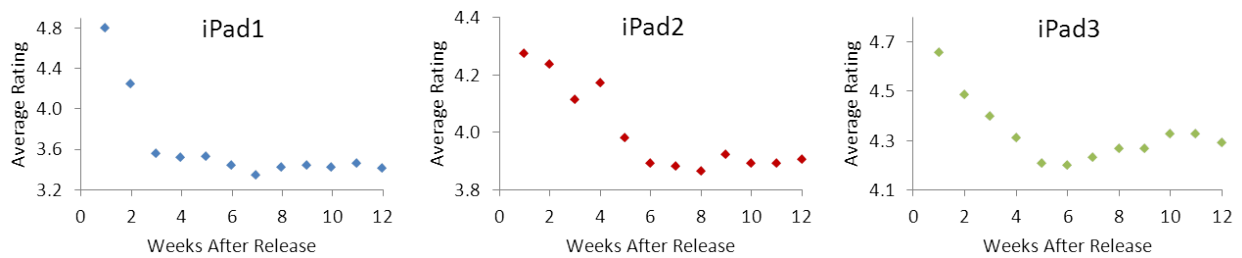


Figure 1 Average rating (cumulative) for Apple iPad1, iPad2 and iPad3 following their introduction in March 2010, 2011 and 2012 respectively. The decreasing average rating suggests that early buyers rated the product more highly on average than subsequent buyers (data collected from Amazon.com via web crawler).

2. Literature Review

This paper proposes a mechanism through which inventory scarcity strategies can be used to influence demand for a newly-introduced product. As such, our work contributes to the literature than lies on the intersection between marketing and operations management. Recent examples of research in this area include Jerath et al. (2010), who study last-minute and opaque selling to strategic consumers; Dai and Jerath (2013), who examine salesforce compensation under inventory considerations; Lobel et al. (2013) and Besbes and Lobel (2014) who investigate the timing of new product launches and dynamic pricing under strategic consumer behavior, respectively.

The notion that scarcity renders products more desirable is intuitively appealing (for work in psychology that focuses on how consumers interpret product scarcity see, for example, Lynn (1992) and Verhallen and Robben (1994)), but it is not always clear why the firm would not simply increase prices or production to alleviate the mismatch between supply and demand. One theory as to why the firm would deliberately induce scarcity is that of “buying frenzies,” first proposed by DeGraba (1995) (see also Courty and Nasiry (2012) for a dynamic model of frenzies). According to this theory, the firm under-produces its product in order to induce consumers to purchase before they become informed of their individual valuations, thus appropriating more consumer surplus. Buying frenzies may occur either when consumers anticipate a future price increase, or when they expect to be rationed if they delay their purchase; none of the two seem to apply to the settings we seek to capture, where the product’s price is typically constant and future availability is ample. Denicolò and Garella (1999) suggest that the firm may find it profitable to restrict early supply when demand rationing is proportional, so as to take advantage of the presence of high-value consumers by increasing price later in the season; this mechanism again relies on prices being dynamic. Another explanation is provided by the signalling-based theory of Stock and Balachander (2005). The authors show that uninformed customers may interpret product scarcity as a signal of high product quality, because this implies that other, better-informed customers chose to purchase. However, the rationale here relies on uninformed customers not being able to observe whether sales to informed customers actually occurred, and not being able to communicate with the informed consumers. These assumptions are plausible in some cases but may be less so in others, particularly for high-profile products that enjoy widespread media coverage. The explanation for deliberate supply shortages proposed in this paper addresses cases where visibility of purchases and communication between the consumer base (e.g., through product reviews) are key features of the setting in consideration.

The consumers’ ability to communicate and learn from each other is the central component of our theory; as such, this paper connects to the literature on SL. In this literature, there are two prevailing approaches to modeling peer-to-peer learning, which may be described as *action-based* and *outcome-based*. Action-based SL typically considers homogeneous agents who receive private *iid* signals regarding the true state of the world (e.g., product quality), and observe the actions of their predecessors (e.g., adoption decisions) before deciding on their own action. In such settings, the seminal papers by Banerjee (1992) and Bikhchandani et al. (1992) demonstrate that herding can be rational, but that rational herds may contain very little information. While earlier work on action-based SL focused on studying the properties of the learning process in the absence of strategic firm considerations, more recent work has incorporated firms’ efforts to modulate the SL process through their decisions. Bose et al. (2006) consider a dynamic-pricing problem in which

the firm uses price as a tool to screen the information transmitted to potential future buyers. For a fixed price, Liu and Schiraldi (2012) find that a sequential (rather than a simultaneous) product launch may benefit firms, since allowing a subset of markets to make observable purchasing decisions before the rest may trigger an adoption herd. Relatedly, Debo and Van Ryzin (2009) demonstrate that an imbalanced allocation of inventory to ex ante identical retailers may help trigger an adoption herd when one of the retailers is seen to stock out.

The SL process in our model differs substantially, in that purchasing actions *in themselves* are not informative because customers share identical ex ante information (i.e., a public prior belief over quality). By contrast, SL occurs on the basis of observable ex post outcomes (i.e., buyers' post-purchase perceptions of product quality), which have become particularly prevalent and influential in the post-Internet era. Godes and Silva (2012) investigate empirically the dynamics of review-generation in online platforms, while Besbes and Scarsini (2013) study theoretically the informativeness of online product ratings when consumers' reports are based not only on their personal experience, but also on how this compares to the product's existing ratings. Given the ever-increasing empirical evidence on the association between buyer reviews and firm profits (e.g., Chevalier and Mayzlin 2006), a growing body of work studies how firms can modulate SL outcomes in online settings through their operational decisions. Ifrach et al. (2011) consider monopoly pricing when buyers report whether their experience was positive or negative, and subsequent customers learn from these reports according to an intuitive non-Bayesian rule (see also Bergemann and Välimäki (1997) who analyze optimal price paths in a duopolistic market). Kuksov and Xie (2010) investigate how pricing should be combined with frills that enhance consumers' product experiences. Jing (2011) and Papanastasiou and Savva (2015) analyze dynamic-pricing policies when consumers may strategically delay their purchase in anticipation of product reviews. The majority of existing research motivated by SL in online settings focuses on the interaction between pricing decisions and the SL process. By contrast, our focus is on decisions pertaining to product availability – the association between quantity decisions and SL outcomes appears to be under-explored in the literature.

While the above papers focus on issues pertaining to “electronic word-of-mouth,” our work also connects to the classic word-of-mouth literature. A distinguishing theme of this work is the attempt to model peer-to-peer learning under cognitive limitations which are commonly encountered in real-world settings. For instance, it may be too cumbersome for agents to incorporate all available information in their decision process, but Ellison and Fudenberg (1993) show that even if consumers use fairly naive learning rules (i.e., “rules-of-thumb”), socially efficient outcomes may occur in the long run. Moreover, it may be costly or impossible for agents to gather all information relevant to their decision, but Ellison and Fudenberg (1995) find that socially efficient outcomes tend to

occur when agents sample the experiences of only a few others. To capture cases in which agents may be likely to observe the experiences of a biased sample of predecessors (e.g., dissatisfied consumers may be more vocal than satisfied consumers), Banerjee and Fudenberg (2004) investigate how the learning process is affected under a range of exogenously specified sampling rules. The quasi-Bayesian model of SL developed in this paper also recognizes the challenges encountered by consumers in the learning process, however, our focus is more on how these challenges affect firm policy, rather than the learning process itself.

Finally, our work also contributes to the literature that investigates the implications for the firm of social factors (other than SL) that influence consumers' decision-making process. Amaldoss and Jain (2005a,b, 2015) consider pricing and branding implications of conspicuous consumption and find experimental evidence of upward-sloping demand curves. Yoganarasimhan (2012) consider a fashion firm's "cloak or flaunt" dilemma when consumers use fashion in an attempt to fit in with their peers but also to differentiate themselves by signaling their good taste. Candogan et al. (2012) analyze pricing strategies when consumers experience positive consumption externalities in a social network. Hu et al. (2013), study inventory implications within a newsvendor model where two substitutable products are sold to consumers whose adoption decisions are directly influenced by those of their predecessors.

3. Setting

We consider a monopolist firm offering a new experiential product to a fixed population of consumers of total mass N . The product is sold over two periods, with the first period representing the product's *launch phase* which is of short time-length. We initially focus on the case where the product's price p is constant across the two selling periods (e.g., Stock and Balachander 2005); we consider dynamic pricing subsequently in §6.3. Each consumer purchases at most one unit of the product throughout the season.

The Consumers Customer i 's gross utility from consuming the product comprises two components, x_i and q_i (e.g., Papanastasiou and Savva 2015, Villas-Boas 2004). Component x_i represents utility derived from product features which are observable before purchase (e.g., product brand, film genre) and is known to the consumer ex ante. We assume that the distribution of x_i components in the population is Normal, $N(\bar{x}, \sigma_x^2)$, with density function denoted by $f(\cdot)$, distribution function $F(\cdot)$ and $\bar{F}(\cdot) := 1 - F(\cdot)$. Component q_i represents utility derived from attributes which are unobservable before purchase (e.g., product usability, actors' performance) and is referred to generically as the product's quality for customer i ; q_i is ex ante unknown to the consumer and is observed only after purchasing and experiencing the product. We assume that the distribution of

ex post quality perceptions is Normal $N(\hat{q}, \sigma_q^2)$, where σ_q captures the overall degree of heterogeneity in consumers' ex post evaluations of the product and \hat{q} is the product's mean quality, which is unobservable to the consumers. The net utility from consuming the product for customer i in either period is defined simply by $x_i + q_i - p$.²

In the first period (i.e., the launch phase), all consumers hold a common prior belief over \hat{q} , which may be shaped, for example, by the firm's advertising efforts, media coverage and pre-release expert reviews.³ This belief is expressed in our model through the Normal random variable \tilde{q}_p , $\tilde{q}_p \sim N(q_p, \sigma_p^2)$; in our analysis, we normalize $q_p = 0$ without loss of generality. We assume that consumers in the first period are willing to purchase a unit of the product provided their expected utility from doing so is positive. Thus, our analysis abstracts away from strategic purchasing delays aimed at acquiring more information about the product (see Papanastasiou and Savva 2015); in §7.1 we demonstrate that explicit consideration of such behavior does not change our results significantly.

In the event that demand outstrips supply (owing to firm-induced scarcity), we assume that excess demand is rationed *efficiently*, meaning that consumers receive a unit in decreasing order of their valuations (e.g., Su 2007, Su and Zhang 2008). Efficient rationing is a natural consequence of consumers competing against each other for limited supply, and is a particularly applicable assumption for cases such as those of Apple product launches, where consumers are seen to form waiting lines leading up to product launch: in Appendix A, we provide (i) a summary of the theory on "waiting-line auctions" by Holt and Sherman (1982) which describes how the formation of waiting lines results in efficient rationing, and (ii) a treatment of the case of proportional rationing of excess demand.⁴ All consumers who secure a unit in the first period report (truthfully) their ex post opinion of product quality q_i to the rest of the market through product reviews.⁵

In the second period, consumers remaining in the market observe the available reviews and update their belief over the product's mean quality from \tilde{q}_p to \tilde{q}_u , via the SL process described in §4. Consumers whose expected utility from purchase is positive given their updated belief purchase

² Since the first period of our model is short, we assume for simplicity that second-period utility is not discounted; this has no qualitative bearing on our results.

³ Dellarocas et al. (2007) find that the correlation between the content of expert and consumer reviews is low, indicating that the two are viewed by consumers as complementary sources of information; they also find that consumer reviews produced shortly after the introduction of a new product are instrumental in predicting its long-term sales trajectory.

⁴ Alternative ways of achieving efficient rationing include the allocation of products on the basis of past consumer purchases or loyalty programs, or the requirement that consumers display the alertness to reserve units in advance of product launch.

⁵ It makes no qualitative difference whether all consumers report their experiences or whether each purchase generates a report with some probability. We do not account "under-reporting bias" as described by Hu et al. (2009), but such behavior would likely lend further support to our proposed scarcity mechanism.

a unit in the second period.⁶

The Firm The firm is assumed to have knowledge of all model parameters through its market research, and seeks to maximize its total expected profit. To focus attention on the association between product availability and SL, we assume that the firm can produce and distribute any quantity of the product in either selling period and incurs zero marginal costs of production. At the beginning of the selling season, the firm chooses the price of the product p . Furthermore, at the beginning of each period the firm decides on the quantity of the product released during that period. The second-period quantity decision is trivial, since the firm will simply fulfill all second-period demand. By contrast, as we will argue in our analysis, the firm may choose to induce scarcity in the first period in order to benefit from the behavioral nature of the SL process. In the context of our model, we define a scarcity strategy as follows.

DEFINITION 1 (SCARCITY STRATEGY). *We say that the firm employs a scarcity strategy when the number of units sold in the first period is smaller than the number of units demanded in the first period.*

Finally, we assume that the firm cannot signal product quality to the consumers through its actions. This assumption is commonly employed in the SL literature (e.g., Ifrach et al. 2011, Sgroi 2002) in order to isolate the effects of peer-to-peer consumer learning. We note that this approach is also supported by existing empirical evidence: for instance, Brown et al. (2012, 2013) observe that consumers in the movie industry do not make quality inferences based on observable firm actions.

4. A Quasi-Bayesian Model of Social Learning

We now describe SL process by which second-period consumers learn from the reviews of first-period buyers. We first present a model of review-generation that allows for dependence of a buyer's review on her idiosyncratic preferences. We then develop a quasi-Bayesian model of learning that incorporates the behavioral feature of SL identified in Li and Hitt (2008).

Review-Generation Each buyer's review consists of her ex post perception of product quality q_i . Building on empirical evidence, we suppose that customer i 's ex post perception of product quality is subject to random noise, but also to a systematic dependence on her idiosyncratic preferences (e.g., Dellarocas et al. 2004, Hu et al. 2009, Li and Hitt 2008). To illustrate, suppose that x_i is the customer's preference for a specific product brand. Intuitively, the same product may be evaluated *differently* by buyers who share an identical preference for the brand (random noise),

⁶Note that the described model implicitly assumes that all consumers in the market are aware of the product's existence from the beginning of the horizon – our model focuses on the informative role of SL (reviews provide information on product performance), as opposed to its role in raising product awareness.

but also by buyers who differ in their brand preference (idiosyncratic). To capture this feature of product reviews, we assume that an individual’s ex post q_i is correlated with her preference component x_i , with correlation parameter ρ . In this case, customer i ’s review follows

$$q_i | x_i = \hat{q} + \rho \frac{\sigma_q}{\sigma_x} (x_i - \bar{x}) + \epsilon_i, \quad (1)$$

where ϵ_i is zero-mean Gaussian noise of variance σ_ϵ^2 , $\sigma_\epsilon^2 = \sigma_q^2(1 - \rho^2)$. When $\rho = 0$, customer i ’s review is simply a random draw from the marginal distribution of ex post quality perceptions $N(\hat{q}, \sigma_q^2)$. By contrast, when $\rho \neq 0$, an individual’s review is a biased draw from $N(\hat{q}, \sigma_q^2)$, with the bias depending on the customer’s idiosyncratic preference component x_i . The sign of ρ determines the nature of the bias, while the magnitude determines its strength. Our analysis focuses on the case $\rho > 0$, where customers with higher-than-average x_i are likely to perceive higher-than-average product quality; for instance if x_i denotes preference for the Apple brand, then we might expect higher- x_i customers to enjoy their experience with the new iPad more than the average consumer.⁷ Alternative assumptions for parameter ρ are discussed in §7.2.

Learning from Reviews The dependance of reviews on preferences poses a challenge for consumers attempting to learn product quality from the reviews of their peers, since although these reviews contain information on \hat{q} , they also reflect opinions which are, to some degree, subjective. The rational approach to learning in this setting would be to “de-bias” each individual review by accounting for the preferences of the reviewer, replace the set of biased reviews with its unbiased counterpart, and then perform the belief update from \tilde{q}_p to \tilde{q}_u by applying Bayes’ rule. Specifically (as we show in Appendix D), after observing a mass of n_1 reviews, generated from buyers whose mean preference component is \bar{x}_1 , and whose average rating is R , the Bayesian posterior belief is Normal $\tilde{q}_u \sim N(q_u, \sigma_u^2)$, with

$$q_u = \frac{n_1 \sigma_p^2}{n_1 \sigma_p^2 + \sigma_q^2 (1 - \rho^2)} \left(R - \rho \sigma_q \frac{\bar{x}_1 - \bar{x}}{\sigma_x} \right) \quad \text{and} \quad \sigma_u^2 = \frac{\sigma_p^2 \sigma_q^2 (1 - \rho^2)}{n_1 \sigma_p^2 + \sigma_q^2 (1 - \rho^2)}. \quad (2)$$

In reality, the above process may be practically inconvenient, or indeed impossible, for two reasons: first, it requires a potentially large amount of cognitive effort; second, it requires perfect information on each reviewer’s characteristics and preferences. Especially in online settings, which are characterized by large numbers of essentially anonymous reviews, both are likely to interfere with consumers’ ability to learn in a fully-rational manner.

Given these difficulties, Li and Hitt (2008) investigate consumer learning empirically and find that consumers take the simple approach of engaging in learning *as if* $\rho = 0$, essentially treating ex

⁷ When $\rho < 0$, customers with higher-than-average x_i are likely to perceive lower-than-average product quality (for instance, Mac users may be more sensitive to software glitches in the new iPad than PC users); extension of our analysis to cases $\rho < 0$ follows in a straightforward manner.

post quality perceptions as unbiased draws from the distribution $N(\hat{q}, \sigma_q^2)$. Whether this approach is the result of a conscious decision to avoid cognitive costs, a heuristic way of dealing with the lack of sufficient information, or simply an erroneous judgement of the review-generating process (or any combination of the three) is unclear. Rather than investigating the causes of this behavior, our goal here is to embed this finding into a theoretical model of SL and test its implications.

Traditionally, behavioral biases in SL have been studied via intuitive heuristic learning rules (proposed as alternatives to Bayes' rule), which are typically custom-designed for the specific setting of interest. More recently, an alternative approach, termed *quasi-Bayesian*, has been proposed – Camerer et al. (2003) remark that they expect “the quasi-Bayesian view will quickly become the standard way of translating the cognitive psychology of judgement into a tractable alternative to Bayes' rule,” while Rabin (2013) discusses its advantages in detail and illustrates the modeling approach through examples from recent literature.

The main idea behind quasi-Bayesian learning models is to depart from Bayes' rule in a controlled manner, and only in the direction of the specific behavioral bias of interest. For example, consumers may have an erroneous understanding of the stochastic process which generates the outcomes they observe, but will otherwise behave as fully-rational agents and apply Bayes' rule in an internally consistent manner (see “warped-model Bayesians” in Rabin (2013); e.g., Barberis et al. (1998), Rabin (2002)). We adopt the warped-model approach and suppose that consumers, for one reason or another, operate under an erroneous internal model of the the stochastic process (1). Specifically, under our proposed quasi-Bayesian model of SL, consumers misrepresent the link between product reviews and reviewer preferences (captured by the correlation parameter ρ) and update their belief from \tilde{q}_p to $\tilde{q}_u \sim N(q_u, \sigma_u^2)$, with

$$q_u = \frac{n_1 \sigma_p^2}{n_1 \sigma_p^2 + \sigma_q^2 (1 - \nu^2)} \left(R - \nu \sigma_q \frac{\bar{x}_1 - \bar{x}}{\sigma_x} \right). \quad (3)$$

Thus, the above learning rule nests both Bayesian updating (case $\nu = \rho$) as well as behavioral updating as documented in Li and Hitt (2008) (case $\nu = 0$). The results of our analysis will apply qualitatively for all cases of $\nu < \rho$, where consumers are prone, at least to some extent, to behavioral biases. However, for ease of exposition, results will be presented for the case of $\nu = 0$; thus, attention can be restricted to the simple learning rule

$$q_u = \frac{n_1 \sigma_p^2}{n_1 \sigma_p^2 + \sigma_q^2} R. \quad (4)$$

Note that the above learning rule retains a number of attractive Bayesian features. The updated mean belief is a weighted average between the prior mean q_p (recall $q_p = 0$) and the average rating

⁸ The posterior variance is given by $\sigma_u^2 = \frac{\sigma_p^2 \sigma_q^2 (1 - \nu^2)}{n_1 \sigma_p^2 + \sigma_q^2 (1 - \nu^2)}$, but is not important for our analysis because consumers are risk-neutral.

from product reviews R . The weight placed on R increases with the number of reviews n_1 (a larger number of reviews renders the average rating more influential), with customers' prior uncertainty over quality σ_p^2 (consumers are more susceptible to reviews when the ex ante quality uncertainty is large), and decreases with the noise in ex post product evaluations σ_q^2 (noise in reviews renders SL less persuasive).

5. Preliminaries

5.1. The Rationale for Scarcity

According to our model setup, the number of reviews generated in the first period is equal to the number (i.e., mass) of first-period purchases. If first-period buyers have an average preference component of \bar{x}_1 , then the average rating R of their reviews follows

$$R = \hat{q} + \rho\sigma_q \frac{\bar{x}_1 - \bar{x}}{\sigma_x} + \epsilon_R, \quad (5)$$

where ϵ_R denotes zero-mean Gaussian noise (i.e., sampling error). Let n_1 denote the number of units sold by the firm in the first period and let $\bar{R} := E[R]$ denote the expected average rating from first-period reviews.

LEMMA 1. *The quantities n_1 and \bar{R} are inversely related; that is, \bar{R} is strictly decreasing in n_1 .*

All proofs are provided in Appendix B. Lemma 1 suggests that by deciding to restrict supply in the launch phase and sell fewer units than demanded, the firm achieves a higher average product rating (albeit with lower review volume). Thus, our model replicates patterns such as those observed in Figure 1, where supply shortages early in the selling season were accompanied by higher average ratings. The mechanism that achieves this effect relies on the combination of efficient rationing (e.g., via a waiting-line allocation as in Apple launches) and preference-dependent reviews (e.g., Dellarocas et al. 2004, Hu et al. 2009, Li and Hitt 2008).

A scarcity strategy that achieves the above effect can be advantageous for the firm, because consumers tend to underestimate the bias in the product's average rating (Li and Hitt 2008). Our quasi-Bayesian model of SL (4) captures this feature of consumer behavior by positing that consumers (in contrast to the fully-Bayesian rule (2)) do not apply the necessary correction to the average rating when engaging in learning from reviews.⁹

⁹ It is a straightforward exercise to show that scarcity cannot be optimal when customers are fully-Bayesian: to see this, note that any bias in the average rating achieved through scarcity is perfectly accounted for by the consumers, and thus offers no advantage to the firm.

5.2. The Firm's Problem

The firm's problem may be expressed as

$$\max_{p, n_1 \leq D_1(p)} p(n_1 + E_R[D_2(p, n_1, R)]), \quad (6)$$

where $D_1(p) := N\bar{F}(p)$ denotes first-period demand for the product at price p (note that any decision $n_1 > D_1(p)$ may be mapped to the decision $n_1 = D_1(p)$ without loss of generality, since any amount of inventory in excess of $D_1(p)$ is leftover in the first period) and D_2 is the number of units demanded and sold in the second period, which depends on the SL outcome through n_1 and R (the functional form of D_2 is made precise in the subsequent sections' proofs). Because problem (6) is not tractable analytically, in our analysis we approximate the firm's problem by

$$\max_{p, n_1 \leq D_1(p)} p(n_1 + D_2(p, n_1, \bar{R})). \quad (7)$$

That is, we solve the firm's problem ignoring the zero-mean error term ϵ_R in (5). As will become evident in our analysis, the firm's decisions are associated with the systematic relationship between n_1 and R described in Lemma 1 (rather than the effects of random sampling error), which is retained by the above simplification. Furthermore, in Appendix C we conduct extensive numerical experiments to demonstrate that the optimal policies and profit functions differ only marginally between problems (6) and (7), for all model parameters considered.

6. Scarcity Strategies

6.1. Exogenous Price

As a first step in our investigation, we examine the role of the launch-quantity decision n_1 in isolation, by solving the firm's problem for an arbitrary and exogenously specified product price p . The case of an exogenous price is practically relevant for cases in which the new product's price is constrained by previous versions of the product or consumers' price expectations (e.g., iPad prices do not tend to differ substantially from one version to the next) – even if the firm cannot price freely, the quantity of the product made available in the launch phase remains a decision variable.

The inverse relationship between n_1 and \bar{R} described in Lemma 1 has a significant impact on the SL process. In particular, defining

$$\bar{q}_u(n_1) = \frac{n_1 \sigma_p^2}{n_1 \sigma_p^2 + \sigma_q^2} \bar{R}(n_1)$$

as the posterior mean belief of consumers remaining in the market conditional on n_1 units being sold in the first period, it appears that the volume of reviews interacts with the average rating in a non-trivial way to shape consumers' updated perception of quality. Under an exogenous price p , the firm's optimal launch-quantity decision is characterized in Proposition 1.

PROPOSITION 1. Let τ^* be the unique solution to the implicit equation

$$\tau = \bar{x} - \frac{\sigma_x \hat{q}}{\sigma_q \rho} + \frac{N \sigma_p^2}{\sigma_q^2} \int_{\tau}^{\infty} (x - \tau) f(x) dx. \quad (8)$$

The following statements hold:

- (i) If $p < \tau^*$, the unique optimal launch quantity is $n_1^* = N\bar{F}(\tau^*) < D_1(p)$.
- (ii) If $p \geq \tau^*$, the unique optimal launch quantity is $n_1^* = D_1(p)$.

The rationale underlying Proposition 1 is as follows. For an exogenous price, the firm's overall profit increases with \bar{q}_u . In the proposition's proof, we show that \bar{q}_u is unimodal in n_1 : too few first-period sales generate a high average rating, but too few reviews to have a significant impact on consumers' perceptions of quality; too many first-period sales generate an impactful but lower average rating. The unimodal property of \bar{q}_u means that, from a SL perspective, there exists for the firm a unique optimal segmentation of the market into higher-valuation *reviewers* and lower-valuation *learners*. This segmentation strikes a balance between the volume and content of launch-phase reviews, and depends on setting characteristics such as the product's quality \hat{q} and consumers susceptibility to persuasion σ_p^2 , among others. When the product's price is below the threshold τ^* , the firm achieves this segmentation by restricting the number of units made available in the first period to $N\bar{F}(\tau^*)$ – this implies a deliberate under-supply of first-period demand for the product (i.e., a scarcity strategy). By contrast, when the product's price is above the threshold τ^* the firm is unable to sell to all desired reviewers in the first period, because some of them are initially unwilling to purchase owing to the product's high price; in such cases, the unimodality of \bar{q}_u suggests that the best feasible approach is to fulfill all demand in the first period.

6.2. Endogenous Price

We now consider the more interesting problem of optimizing both the product's price and the launch-phase quantity simultaneously. We first establish a necessary and sufficient condition for product scarcity to be beneficial for the firm, and show that this can be expressed as a simple condition on the product's quality \hat{q} . Next, we provide guidelines regarding the optimal implementation of scarcity strategies and how this depends on product and market characteristics. We then consider the implications of induced shortages for total welfare and consumer surplus.

The result of Proposition 1 is particularly useful in gaining insight into the firm's optimal pricing-and-quantity policy. Specifically, given a set of product and consumer characteristics, Proposition 1 suggests that the firm may choose to employ one of two types of policy. The first type entails relatively lower prices combined with a restricted number of units made available in the launch phase (cases $p < \tau^*$ in Proposition 1). The second type employs relatively higher prices combined with unlimited first-period product availability (cases $p > \tau^*$ in Proposition 1). While the first

type achieves a better SL outcome than the second, the second type generates higher revenue-per-purchase than the first. The following result provides a necessary and sufficient condition for scarcity to form part of the firm's optimal policy.

PROPOSITION 2. *A scarcity strategy is optimal if and only if*

$$\tau^* h(\tau^* - \bar{q}_u^*) > 1,$$

where $h(\cdot) := \frac{f(\cdot)}{F(\cdot)}$ and $\bar{q}_u^* = \max_{n_1 \in [0, N]} \bar{q}_u(n_1)$.

Proposition 2 fully characterizes the region of the parameter space where the firm optimally chooses to induce product scarcity in the first period. The interaction between these parameters is complex, but the most significant implication of the above result is the existence of a threshold on the product's quality \hat{q} that determines whether a scarcity strategy is profitable.

COROLLARY 1. *There exists a threshold Q such that:*

- (i) *If $\hat{q} < Q$, a scarcity strategy is optimal. The unique optimal pricing-and-quantity policy is described by $\{p^*, n_1^*\} = \{\xi, m\}$, where ξ satisfies $\xi h(\xi - \bar{q}_u(m)) = 1$ and $m = N\bar{F}(\tau^*) < D_1(\xi)$.*
- (ii) *If $\hat{q} \geq Q$, a scarcity strategy is not optimal. The optimal pricing-quantity policy is described by $\{p^*, n_1^*\} = \{\zeta, D_1(\zeta)\}$, where $\zeta = \arg \max p\bar{F}(p - \bar{q}_u(D_1(p)))$.*

The value of the threshold Q is typically high with respect to the consumers' prior expectation of quality q_p – roughly speaking, a scarcity strategy benefits the firm unless the product's quality is unexpectedly high. To illustrate, we provide in Figure 4 region plots of parameter combinations where scarcity is optimal; in the majority of cases, we observe that the quality threshold is such that $Q > q_p + \sigma_p$.

When product quality is below the threshold Q , the firm's chosen policy is oriented towards optimizing the SL process. The firm sets a price which is relatively affordable ($p^* < \tau^*$), however, it does not allow all consumers who wish to purchase a unit in the launch phase to do so; instead, it uses its quantity decision to optimally segment the market into reviewers and learners. With the help of the optimized SL process, it then achieves a high volume of sales in the second period. By contrast, when quality is above the threshold Q , the firm takes a very different approach. It sets a high price ($p^* > \tau^*$) so that only a small number of consumers are willing to purchase in the first period, and ensures ample product availability so that all such consumers obtain a unit in the launch phase. The firm then relies on the product's inherently high quality to drive second-period sales, rather than an optimized SL process (which would entail lower revenue-per-purchase) – the result is a moderate number of high-revenue purchases.

For the remainder of this section, we will focus on regions of the parameter space where scarcity is optimal. We describe first the dependance of the optimal price-and-quantity policy, as well as the firm's profit, on market and product characteristics.

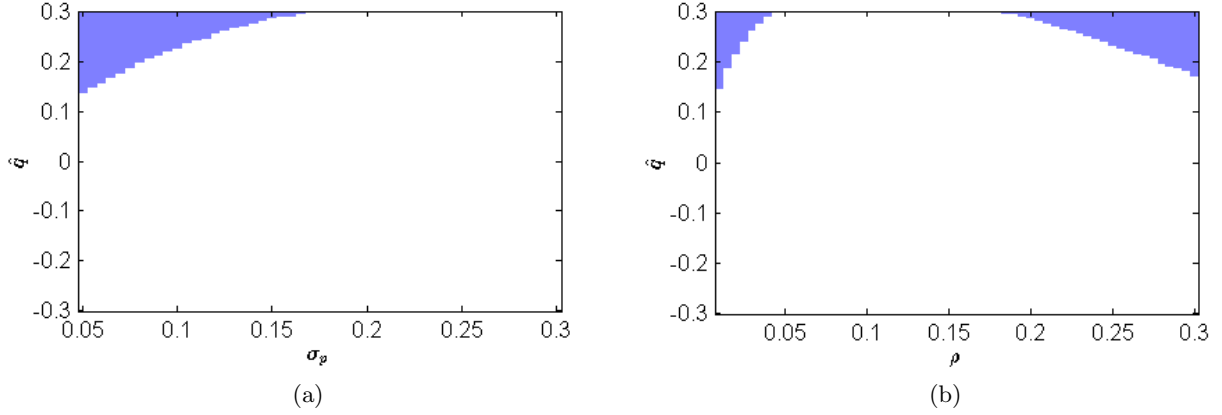


Figure 2 Region plots: white regions mark the optimality of a scarcity strategy.

Parameter values: $N = 1000$, $\bar{x} = 0.5$, $\sigma_x = 0.25$, $q_p = 0$, $\sigma_p = 0.2$, $\sigma_q = 1$, $\rho = 0.15$.

PROPOSITION 3. *The optimal scarcity strategy admits the following properties:*

- (i) *The optimal price p^* is strictly increasing in the product's quality \hat{q} , the correlation coefficient ρ , and customers' prior uncertainty σ_p .*
- (ii) *The optimal launch quantity n_1^* is strictly increasing in \hat{q} , strictly decreasing in σ_p , and strictly increasing (decreasing) in ρ for $\hat{q} \geq 0$ ($\hat{q} < 0$).*
- (iii) *The firm's optimal profit π^* is strictly increasing in \hat{q} , ρ , σ_p .*

Intuitively, the firm will charge a higher price for products of higher quality, products for which high-valuation consumers are highly biased in their opinions, and products for which subsequent consumers are more uncertain of quality, and therefore more susceptible to the opinions of the early buyers. The properties of the optimal launch quantity are less intuitive, and are associated with the optimal segmentation of the market described in §6.1. For a fixed n_1 , an increase in either \hat{q} or ρ results in an increased average rating \bar{R} (see (5)). Interestingly, increases in the two parameters call for different types of adjustment in the firm's launch quantity: as \hat{q} increases, the firm always prefers to increase the launch quantity so that consumers place more weight on the average rating when performing their update; by contrast, as ρ increases, the firm's adjustment of the launch quantity is contingent on the product's quality: if quality is higher than consumers' prior expectations, then the firm increases the launch quantity; if quality is lower than consumers' prior expectations, then the firm reduces the launch quantity with the goal of extracting more favorable review content \bar{R} . Finally, the more uncertain consumers are about the product's quality, the fewer the units that are made available by the firm in the launch phase; in this case, just a small number of reviews is enough to significantly influence consumers' opinions of the product, and the firm therefore chooses to focus on review content rather than volume.

Proposition 3 also allows for a comparison of the firm's optimal policy and profit against two benchmarks cases. The first is one where consumers do not engage in SL; in our model, this case

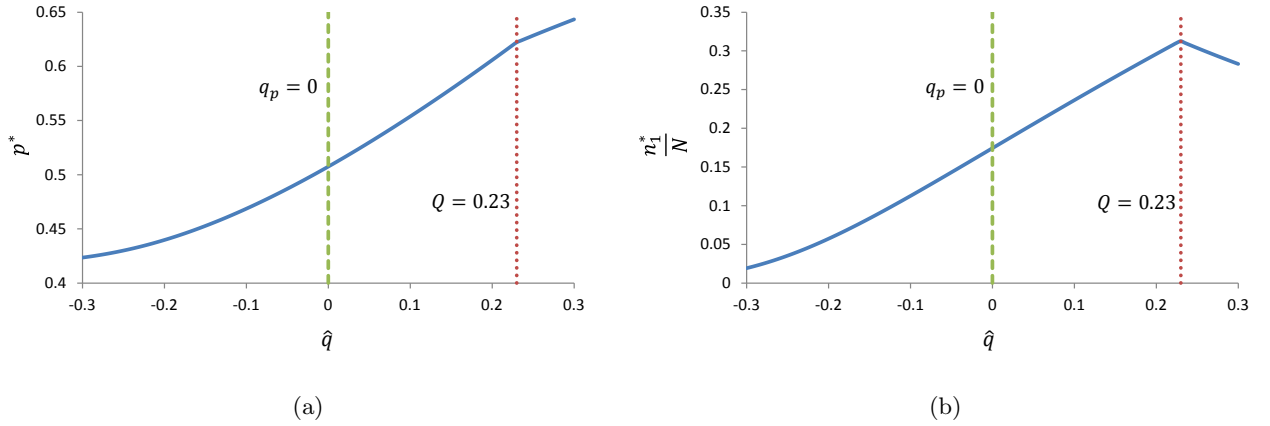


Figure 3 Optimal pricing-quantity policies at different values of product quality \hat{q} : (a) Optimal price p^* (b) Optimal launch quantity (normalized by total market size) $\frac{n_1^*}{N}$. Parameter values: $N = 1000$, $\bar{x} = 0.5$, $\sigma_x = 0.25$, $q_p = 0$, $\sigma_p = 0.1$, $\sigma_q = 1$, $\rho = 0.15$.

is operationally equivalent to the case $\sigma_p \rightarrow 0$, because this entails that consumers' updated mean belief in the second period is equal to the prior mean belief (see (4)). The second benchmark is one where consumers' post-purchase opinions are unbiased (i.e., are not influenced by their preferences); this corresponds to the case $\rho = 0$ in our general model (note that this benchmark is also qualitatively equivalent to the case in which reviews remain biased, but consumer learning is fully-Bayesian, i.e., $\nu = \rho > 0$ in (3)). In comparison with both benchmark cases, we find that the firm in the general case of our model (a) sets a higher price, (b) makes fewer units available in the launch phase, and (c) achieves higher overall profit.

More generally, we point out that the firm's profit in the absence of SL relies exclusively on the belief with which consumers enter the market. By contrast, in the presence of SL the firm's profit depends less on this prior belief, and more on the SL process. This observation marks a more general shift in the activities of the modern-day firm: while in times past firms focused more on promotional efforts aimed towards influence consumers' beliefs (i.e., consumers' prior when there is no SL in our model), there is now increasing investment in promoting and optimizing the SL process (e.g., KIA Motors have recently launched an advertising campaign aimed towards promoting the reviews of satisfied customers; see "Kia Reviews and Recommendations").

To conclude this section, we conduct a numerical investigation into the implications of induced supply shortages for consumer surplus and total welfare. By restricting supply in the launch phase, the firm leverages the idiosyncratic preferences of early buyers to generate a more favorable SL effect, thus increasing overall product adoption and profit. Importantly, supply shortages amplify the bias in review content (this manifests as a higher average rating), which translates into (some) "erroneous" customer decisions in the second period: had the information contained in product reviews been unbiased, fewer consumers would have chosen to purchase in the second period.

Presumably, policies involving supply shortages therefore result in a decrease in the consumers' surplus; furthermore, if the loss in consumer surplus is larger than the firm's gain in profit, then supply shortages will be total-welfare-decreasing.

To make the above discussion more precise, we compare consumer surplus and total welfare when the firm employs the pricing and inventory policy $\{p^*, n_1^*\}$ described in Corollary 1 versus that when the firm chooses the product's price p_n , but satisfies all first-period demand (i.e., releases at least $\bar{F}(p_n)$ units in the first period). More specifically, since all consumers with $x_i > \underline{x}_b := p - q_u(n_1) = p - \frac{n_1 \sigma_p^2}{n_1 \sigma_p^2 + \sigma_q^2} \hat{q} - \rho \frac{\sigma_q}{\sigma_x} (\bar{x}_1 - \bar{x})$ purchase the product in either the first or the second period, the ex-ante consumer surplus when the firm charges price p and releases n units of inventory is given by

$$S(p, n) = E_{\hat{q}} \left[\int_{p - q_u(n)}^{\infty} \left(x_i + \hat{q} + \rho \frac{\sigma_q}{\sigma_x} (x_i - \bar{x}) - p \right) f(x) dx \right],$$

and, since the price p paid by customers is a transfer to the firm, the total welfare is given by

$$W(p, n) = E_{\hat{q}} \left[\int_{p - q_u(n)}^{\infty} \left(x_i + \hat{q} + \rho \frac{\sigma_q}{\sigma_x} (x_i - \bar{x}) \right) f(x) dx \right].$$

We also note that only a fraction of these consumers, those with $x_i > \underline{x}_a := \frac{\rho \frac{\sigma_q}{\sigma_x} \bar{x} + p - \hat{q}}{\rho \frac{\sigma_q}{\sigma_x} + 1}$, enjoy a positive surplus. For those customers (if any) for whom $\underline{x}_b \leq x_i < \underline{x}_a$, the surplus from consuming the product is negative – these are the customers who are over-influenced by the SL process and make an “erroneous” purchasing decision. Given the analytically intractable optimal pricing/inventory policy is not possible to investigate how the consumer/total welfare changes analytically. However, in Table 1, we present the difference in total welfare ($\Delta W = W(p^*, n_1^*) - W(p_n, \bar{F}(p_n))$) and consumer surplus ($\Delta S = S(p^*, n_1^*) - S(p_n, \bar{F}(p_n))$) between the two policy types, for different combinations of our model parameters; a positive difference indicates that W or S is higher under policies involving scarcity. We note that our calculations do not include welfare losses associated with the rationing process, which we do not explicitly model.

There are two main observations. First, the impact of policies involving supply shortages on total welfare is positive (i.e., $\Delta W > 0$) under all parameter combinations. This implies that any losses in consumer surplus, which occur as a result of the firm's manipulation of the SL process, are more than compensated for by the increase in the firm's profit. Second, surprisingly, we observe that supply shortages in many cases constitute a “win-win” situation; that is, shortages often generate an increase not only in the firm's profit, but also in the consumers' surplus. A closer look (see Figure 4) reveals that shortages are beneficial for both the firm and the consumers when the correlation ρ is low and the heterogeneity in consumers' preference σ_x is high. When ρ is low, the SL rule employed by consumers approaches Bayesian updating, therefore resulting in fewer erroneous decisions. But even when ρ is relatively high, we observe that consumers may be better

\hat{q}	-0.15	-0.1	-0.05	0	0.05	0.1	0.15
$\Delta W > 0$	100%	100%	100%	100%	100%	100%	100%
$\Delta S > 0$	3%	10%	22%	37%	48%	60%	72%

Table 1 Impact of supply shortages on Total Welfare and Consumer Surplus: $\Delta W > 0$ represents an increase in Total Welfare, $\Delta S > 0$ represents an increase in Consumer Surplus. Scenario parameters:

$N \in \{1000, 3000, 5000, 7000\}$; $q_p = 0$, $\sigma_p = 0.1$, $\hat{q} \in \{-0.15, -0.1, -0.05, 0, 0.05, 0.1, 0.15\}$; $\sigma_x \in \{0.4, 0.6, 0.8\}$, $\sigma_q = 0.6$, $\bar{x} = 0.5$; $\rho \in \{0.1, 0.15, 0.2, 0.25, 0.3\}$.

off under shortage policies provided they are sufficiently heterogeneous in their preferences – this is surprising, particularly since preference heterogeneity is a main contributor to the bias in the SL process.

The reasoning here is as follows. When the firm fulfills all demand in the launch phase, it typically chooses a relatively high price: the pricing decision represents a compromise between achieving a “good” SL effect (by charging a high price and therefore selling to relatively high-valuation consumers in the first period) and keeping the price low enough to drive a satisfactory amount of overall sales. By contrast, when the firm optimizes the launch-phase quantity decision, the pricing decision is decoupled from the SL process: the firm modulates the SL outcome through the quantity decision, and sets the price at a lower level which favours wider product adoption. In the latter case, although supply shortages amplify the bias in the review content, at the same time they may leave consumers who purchase the product with higher, on average, surplus, despite the fact that some consumers experience ex post negative outcomes. The consumers population is therefore more likely to benefit from supply shortages when the difference in price between the two policies is higher. As Figure 4 suggests, this occurs when consumers preferences are more heterogeneous, because in this case consumers’ valuations for the product are more diffuse.

6.3. Dynamic Pricing

We now consider the case where the firm is able to charge a different price in the product’s launch-phase (we note, however, that in many practical settings such a strategy can be difficult to implement, for example, due to fairness considerations; e.g., *The New York Times* (2007)). Let p_1 and p_2 denote the first- and second-period price, respectively. Our first result suggests that the ability to price dynamically completely nullifies the benefit of induced product scarcity.

PROPOSITION 4. *Let $\{p_1^*, p_2^*\}$ be the optimal dynamic-price plan and let n_{1d}^* be the corresponding optimal launch-phase quantity. Then $n_{1d}^* = D_1(p_1^*)$.*

In the case of fixed pricing, by restricting supply to the optimal quantity described in Corollary 1, the firm was able to achieve the optimal balance between the volume of product reviews and the

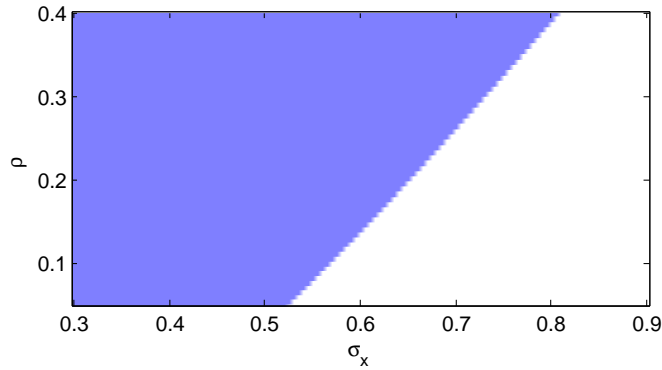


Figure 4 Impact of supply shortages on consumer surplus: white (shaded) regions mark $\Delta S > 0$ ($\Delta S < 0$).

Parameter values: $N = 3000$, $q_p = 0$, $\sigma_p = 0.1$, $\hat{q} = 0$, $\sigma_q = 0.6$, $\bar{x} = 0.5$.

content of these reviews. However, this balance can also be achieved by attaching an appropriate price-premium to first-period purchases since, in this case, the consumers from which the firm wishes to extract reviews will be the only ones who are willing to purchase a unit in the first period. In this way, the firm not only achieves the optimal reviewer-learner segmentation in the market, it also extracts increased surplus from high-valuation consumers. As a result, any fixed-price-with-scarcity strategy is dominated by some dynamic-pricing strategy where product availability is ample.¹⁰

The result of Proposition 4 is interesting when compared against existing theories regarding the optimality of early firm-induced supply shortages. For instance, DeGraba (1995) and Courty and Nasiry (2012) describe how a monopolist may create an early supply shortage so as to induce a “buying frenzy” among consumers, while Denicolò and Garella (1999) argue that early shortages can be optimal provided rationing is not efficient. Crucially, such theories have dynamic pricing as a requirement and do not apply to settings in which the product’s price remains unchanged after the launch phase. By contrast, supply shortages in our setting are optimal *only* under a constant price. The mechanism underlying the profitability of shortages in our paper is, of course, fundamentally different from those in existing theories, which do not consider SL among consumers; nevertheless, our results complement the existing literature in the sense that they advocate the optimality of early supply shortages in distinctly different settings, in which fixed pricing is commonplace.

Proposition 4 also suggests that, under fixed pricing, the decision to restrict supply in the first period may be viewed as an indirect method of price-discrimination. The next question we

¹⁰ Keeping in line with the rest of our analysis, the result of Proposition 4 assumes that consumers do not strategically delay their purchases; this does not affect the result or the underlying rationale, and allows for a fair comparison between scarcity strategies and dynamic pricing.

consider is how the firm's profit under this indirect method of price-discrimination (i.e., fixed price with scarcity) compares with that under direct price-discrimination (i.e., dynamic pricing). The difference in the firm's profit under the two strategies is characterized in Proposition 5.

PROPOSITION 5. *Let π_s^* denote firm profit under the optimal fixed-pricing policy $\{p^*, n_1^*\}$ with $n_1^* \leq D_1(p^*)$, and let π_d^* denote firm profit under the optimal dynamic pricing policy $\{p_1^*, p_2^*\}$ with $p_1^* \geq p_2^*$. Then the difference in profits between the two policies $\Delta\pi^* = \pi_d^* - \pi_s^*$ is bounded by*

$$(\tau^* - p^*)n_1^* \leq \Delta\pi^* \leq (p_1^* - p_2^*)D_1(p_1^*), \quad (9)$$

where τ^* is the solution to (8).

The intuition for the bounds in Proposition 5 is as follows. First, consider the lower bound. Under fixed pricing, firm profit is maximized at $\{p^*, n_1^*\}$. When employing dynamic pricing, the monopolist can at least charge $p_1 = \tau^*$ and $p_2 = p^*$ and extract profit which exceeds its profit under fixed pricing by $(\tau^* - p^*)n_1^*$; the excess profit is generated through the price-premium attached to first period purchases. Next, consider the upper bound. Under dynamic pricing, the monopolist achieves maximum profit at $\{p_1^*, p_2^*\}$ (recall from Proposition 4 that the optimal quantity decision is $n_{1d}^* = D_1(p_1^*)$). When employing fixed pricing, the monopolist can at least set $p = p_2^*$ and $n_1 = D_1(p_1^*)$. In this way, the firm forgoes only the revenues extracted through the first period price premium under dynamic pricing, which is the expression of the upper bound.

To complement the result of Proposition 5, we conducted numerical experiments to examine the effectiveness of optimizing the launch-phase quantity decision. Our numerical experiments indicate that not only do early supply shortages provide the firm with a profit advantage over policies with unrestricted supply, in many cases they allow the firm to retrieve most of the profit lost due to the firm's inability to price dynamically in the launch-phase; an example is presented in Figure 5. Across all combinations of parameters listed in Table 1, we find that the profit gap between dynamic pricing and fixed pricing *without* shortages is 23% on average with a minimum gap of 11% (maximum 35%), while the gap between dynamic pricing and fixed pricing *with* supply shortages is 13% on average with a minimum of 3% (maximum 24%). Overall, we observe that policies involving supply shortages generate substantial value for the firm when product quality is relatively close to customers' prior expectations (e.g., see Figure 5b).

7. Extensions

7.1. Strategic Purchasing Delays

When the product's price is fixed across time, consumers have no monetary incentive to strategically delay their purchase in the first period. However, provided consumers are sufficiently patient, strategic delays may occur for informational reasons; that is, some consumers with a positive

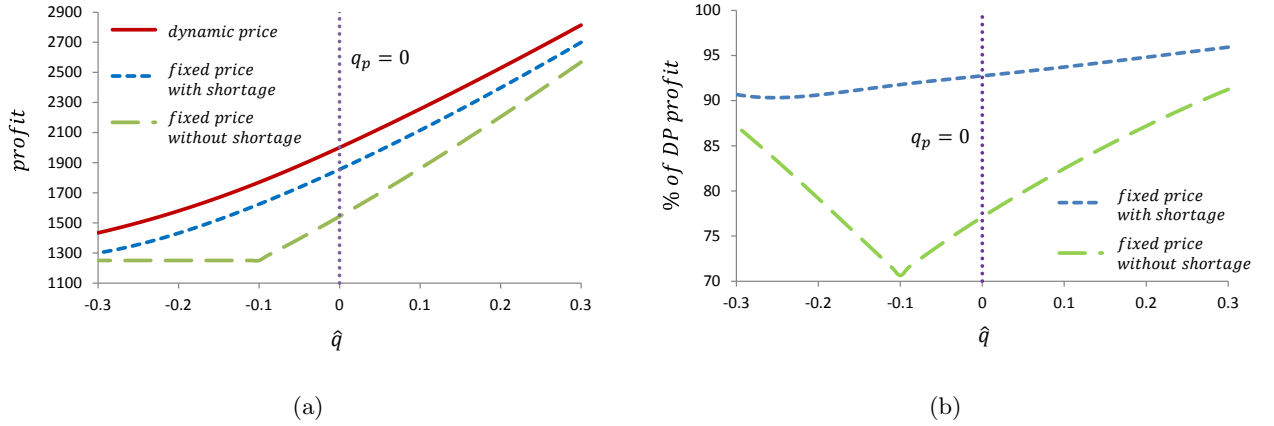


Figure 5 (a) Firm profit under different policies (b) Percentage of dynamic-pricing profit achieved under fixed-pricing policies. Parameter values: $N = 5000$, $\bar{x} = 0.5$, $\sigma_x = 0.4$, $q_p = 0$, $\sigma_p = 0.1$, $\sigma_q = 0.6$, $\rho = 0.2$.

expected utility from purchase in the first period may choose to delay their purchasing decision in anticipation of the information contained in the reviews of their peers (see Papanastasiou and Savva (2015)). Here, we illustrate that such consumer behavior has no significant bearing on our analysis.

Let us first make the concept of strategic waiting more concrete, by introducing a parameter δ_c ($0 \leq \delta_c \leq 1$), which represents consumers' patience or "strategicness" – consumers are myopic when $\delta_c = 0$ and become more strategic as δ_c increases (Cachon and Swinney 2011). Parameter δ_c enters consumers' second-period utility as a multiplicative discount factor, and each consumer seeks to maximize her expected utility through her purchasing decisions. The analysis of Papanastasiou and Savva (2015) indicates that strategic consumers will follow a threshold policy in the first period, whereby consumers purchase in the first period provided $x_i > \theta(p) > p$; that is, consumers whose expected utility is relatively high (low) in the first period choose to purchase immediately (delay purchase). Intuitively, the threshold $\theta(p)$ is strictly increasing in price p and in consumers' patience δ_c .

Next, consider how the result of Proposition 1 is affected by strategic purchasing delays. If the price of the product satisfies $p > \tau^*$, in the absence of strategic delays the firm prefers to sell as many units as possible in the first period. This remains the case in the presence of delays, the difference being that the firm will now be able to sell only $D_1(\theta(p))$, as opposed to $D_1(p)$ units, in the first period. On the other hand, if price satisfies $p \leq \tau^*$ then in the absence of strategic delays it is optimal for the firm to sell only $D_1(\tau^*)$ units in the first period. When consumers are strategic, two possible cases arise: either (i) $D_1(\theta(p)) > D_1(\tau^*)$, in which case the firm continues to under-supply the launch-phase demand (which is now equal to $D_1(\theta(p))$) as opposed to $D_1(p)$ and makes only $D_1(\tau^*)$ units available in the first period, or (ii) $D_1(\theta(p)) \leq D_1(\tau^*)$, in which case the firm sells as many units as possible in the first period, namely, $D_1(\theta(p))$. Case (i) applies to

relatively lower values of price, while case (ii) applies to values closer to (but still lower than) τ^* . The end result is simple: in the presence of strategic purchasing delays, there exists again a threshold price below which the firm deliberately under-supplies early demand, say ψ^* , but this threshold will be lower than τ^* as a result of strategic consumer behavior (i.e., for $\delta_c > 0$, we have $\psi^* < \tau^*$). Furthermore, when $p < \psi^*$ the optimal first-period quantity remains $D_1(\tau^*)$, as was the case without strategic delays – in terms of the formal statement of Proposition 1, the only thing that changes is the threshold on the product’s price. Since Proposition 1 is the foundation for our subsequent results, it follows that our main results and model insights are qualitatively unchanged by the presence of strategic purchasing delays.

7.2. Quality-Dependent Correlation ρ

In our main analysis, we have focused on a positive correlation $\rho > 0$ between consumer preferences and ex post quality perceptions (Dellarocas et al. 2004, Li and Hitt 2008). A plausible alternative assumption would be to assume that the value of the correlation parameter depends on how the consumers’ ex post experiences compare with their ex ante expectations. One way of implementing this assumption is to make ρ a function of the comparison between the product’s true mean quality \hat{q} and the consumers’ prior expectation q_p . Consider the following simple case:

$$\rho = \begin{cases} +k & \text{if } \hat{q} \geq q_p \\ -k & \text{if } \hat{q} < q_p, \end{cases}$$

for some positive constant k . That is, if $\hat{q} \geq q_p$ then the product’s quality exceeds consumers’ expectations and reviews are positively biased, while if $\hat{q} < q_p$ then quality is below expectations and reviews are negatively biased. Under this structure for ρ , our main analysis holds unchanged for cases of $\hat{q} \geq q_p$ (i.e., we can simply apply $\rho = k$ in our existing results). When $\hat{q} < q_p$, consumers’ opinions will be negatively biased. In this case, it is straightforward to show that the firm will choose the single-period optimal price, fulfill all demand in the first period, and achieve zero sales in the second period.

8. Conclusion

Motivated by empirical evidence from consumer learning in online settings, as well as anecdotal observations regarding the impact of early short-term supply shortages on consumer reviews, this paper proposes and investigates a SL-based explanation for the optimality of early firm-induced supply shortages. Whether this theory has more explanatory power over existing theories is ultimately an empirical question, but we note that there are several settings where the existing theories do not apply, namely, where prices are fixed and consumer peer-to-peer communication is prominent. Interestingly, even though supply shortages may be viewed as an attempt by the firm to

manipulate the SL process, in many cases they result in “win-win” situations, whereby both the firm and the consumer population are left better off in terms of profit and surplus, respectively.

Our findings have implications that are relevant for both marketing and operations management. First, by optimizing the launch-phase quantity decision, we find that firms may be able to enjoy most of the profit-benefits of pricing dynamically while charging a fixed price. In this respect, inventory management can be viewed as a subtle and effective substitute for dynamic pricing, in cases in which the latter is difficult to implement. Second, the increase in firm profit associated with SL from buyer reviews explains why firms have invested in online systems that allow their customers to post reviews and interact. Third, our model also indicates that firms may benefit from aiming their pre-launch advertising campaigns at customers who are predisposed to having a positive experience with their product. In terms of global product launches, firms which are aware of a significant pool of loyal customers in specific markets may wish to consider launching their product first in such favorable regions, in order to extract higher product ratings. Fourth, the optimality of early supply shortages suggests that early non-deliberate stock outs may not be as detrimental as previously considered (e.g., see Ho et al. (2002), Ho et al. (2011)), provided the firm facilitates efficient rationing of demand through appropriate rationing mechanisms (e.g., waiting-lines, loyalty programs).

Appendix

A. Supplementary Materials

A.1. Efficient Rationing via Waiting Lines

In this section, we discuss how efficient rationing occurs when new products are allocated via waiting lines, whereby consumers may “queue-up” in advance of product launch and receive a unit on a first-in-line first-served basis (e.g., Apple product launches; *The Los Angeles Times* (2011)). (Note that consumers may become aware of short supply through firm announcements (*TechNewsWorld* 2006), media speculation (*The Washington Post* 2012), or prior experience with similar products.)

In this setting, customers who choose to join the waiting-line relatively earlier have a higher chance of securing a unit, but must spend a larger amount of costly time in the waiting line. Thus, customers who wish to purchase a unit in the first period are viewed as participants in an auction, in which bids for the product are made in units of costly waiting-time spent in the waiting line. As demonstrated by Holt and Sherman (1982), the composition of the waiting-line that forms is an equilibrium outcome that depends on the relationship between consumers’ valuations for the product and their waiting cost per unit time. Let w_i denote customer i ’s waiting cost per unit time and let x_i and w_i be related through the mapping $w_i = w(x_i)$, where $w : \mathbb{R} \mapsto \mathbb{R}$ and $w(\cdot)$ is assumed to be differentiable across its domain. Adapting the analysis of Holt and Sherman (1982) to our setting we may state the following result.

PROPOSITION 6 (Holt and Sherman (1982)). *If $\eta = \frac{(x_i - p)w'(x_i)}{w(x_i)} < 1$, then in equilibrium customers with $x_i \geq p$ join the first-period waiting line in descending order of their x_i components.*

Under the sufficient condition of Proposition 6, customers holding relatively higher ex ante valuations for the product will choose to queue-up relatively earlier; in this case, the composition of the waiting-line results in efficient rationing. We note that the sufficient condition of Proposition 6 subsumes both the commonly assumed case of homogeneous waiting costs, as well as the case of higher valuation customers incurring relatively lower waiting costs, which appears to be consistent with anecdotal evidence that those who join the waiting line first are high-valuation, low-cost-of-waiting customers such as students (UTV 2012) or speculators. (The presence of speculators in the market does not affect the applicability of our model, since speculators may be viewed as an additional channel through which units are allocated to high-valuation customers.)

A.2. Proportional Rationing

In this section, we illustrate that when demand rationing in the launch phase is proportional (i.e., each customer who is willing to purchase in the first period has an equal chance of securing a unit), then early scarcity strategies cannot be optimal for the firm. Proportional rationing occurs, for example, when the firm allocates the available units via a lottery system (e.g., ZDNet (2012)). Under proportional rationing, the following result suggests that the firm maximizes its profit by ensuring ample first-period product availability.

PROPOSITION 7. *For any price p and any $\rho > 0$, if first-period rationing is proportional then it is optimal for the firm to sell as many units as possible in the launch phase, that is, $n_1^* = D_1(p)$.*

In accordance with conventional knowledge, the more the firm sells in the first period, the higher its overall profit. The sub-optimality of any decision $n_1 < D_1(p)$ for the firm is explained as follows. Under proportional rationing, the average preference component of first-period buyers (\bar{x}_1 in (5)) is fixed and independent of n_1 . As a result, the average rating \bar{R} is also fixed and independent of n_1 ($\bar{R} = \hat{q} + \rho\sigma_q\sigma_x h(p)$, where $h(\cdot) := \frac{f(\cdot)}{1-F(\cdot)}$). In this case, the decision n_1 modulates only the weight placed by consumers on \bar{R} when updating their beliefs (see (4)).

Depending on \hat{q} and the value of ρ , the SL process may either increase or decrease the valuations of consumers remaining in the market; however, in both cases it is optimal for the firm to sell as many units as possible in the first period. If $\bar{R}(p) > q_p = 0$, then the firm sells as many units as possible in the first period, because a larger number of “good” reviews will generate larger demand in the second period by increasing consumers’ posterior belief over quality. By contrast, if $\bar{R}(p) < 0$, then reviews are “bad” for the firm, because they decrease consumers’ perception of quality. Nevertheless, it is still optimal for the firm to sell as many units as possible in the first period, for if it does not, it loses sales in the second period through the effects of SL.

B. Proofs

Proof of Lemma 1

Let $\tau(n_1) := \bar{F}^{-1}\left(\frac{n_1}{N}\right)$. Under efficient rationing, we have

$$\bar{R}(n_1) = \hat{q} + \rho\sigma_q \frac{\bar{x}_1 - \bar{x}}{\sigma_x} = \hat{q} + \rho\sigma_q \frac{E[x_i | x_i > \tau(n_1)] - \bar{x}}{\sigma_x} = \hat{q} + \rho\sigma_q \frac{\frac{\int_{\tau(n_1)}^{+\infty} x f_1(x) dx}{\bar{F}(\tau(n_1))} - \bar{x}}{\sigma_x}.$$

Note that the truncated Normal mean $\frac{\int_{\tau}^{+\infty} x f_1(x) dx}{\bar{F}(\tau)}$ is strictly increasing in τ , and that $\tau(n_1)$ is strictly decreasing in n_1 . Therefore, \bar{R} is strictly decreasing in n_1 .

Proof of Proposition 1

Under efficient rationing, the firm's problem may be stated as $\max_{n_1 \leq D_1(p)} n_1 p + \max\{(N\bar{F}(p - \bar{q}_u) - n_1), 0\}p$ where the two terms correspond to first- and second-period profit respectively (note $D_2 = \max\{(N\bar{F}(p - \bar{q}_u) - n_1), 0\}$). Equivalently, the firm's problem is $\max_{n_1 \leq D_1(p)} \max\{N\bar{F}(p - \bar{q}_u), n_1\}p$. Because the firm always has the option to choose $n_1 = 0$ so that $\bar{q}_u = q_p = 0$ and achieve $N\bar{F}(p)$ sales in the second period, it suffices to consider the problem $\max_{n_1 \leq D_1(p)} N\bar{F}(p - \bar{q}_u)p$ and, since $\bar{F}(p - \bar{q}_u)$ is increasing in \bar{q}_u , the optimization problem simplifies to $\max_{n_1 \leq D_1(p)} \bar{q}_u$. We next consider the properties of the function $\bar{q}_u(n_1)$. We have $\bar{q}_u(n_1) = \frac{n_1 \sigma_p^2}{n_1 \sigma_p^2 + \sigma_q^2} \bar{R}(n_1)$, and

$$\bar{R}(n_1) = \hat{q} + \rho\sigma_q \frac{E[x_i | x_i > \tau(n_1)] - \bar{x}}{\sigma_x},$$

where $\tau(n_1)$ is given by $P(x_i \geq \tau) = \bar{F}(\tau) = \frac{n_1}{N}$. Using this expression we have

$$\begin{aligned} \bar{q}_u(\tau) &= \frac{N\bar{F}(\tau)\sigma_p^2}{N\bar{F}(\tau)\sigma_p^2 + \sigma_q^2} \left[\hat{q} + \rho\sigma_q \frac{\frac{\int_{\tau}^{+\infty} x f_1(x) dx}{\bar{F}(\tau)} - \bar{x}}{\sigma_x} \right] \\ &= \frac{N\sigma_p^2}{N\bar{F}(\tau)\sigma_p^2 + \sigma_q^2} \left[\bar{F}(\tau)\hat{q} + \rho\sigma_q \frac{\int_{\tau}^{+\infty} x f_1(x) dx - \bar{F}(\tau)\bar{x}}{\sigma_x} \right]. \end{aligned}$$

Differentiating with respect to τ , we have (after some manipulation)

$$\frac{\partial \bar{q}_u}{\partial \tau} = \frac{Nf(\tau)\sigma_p^2}{[N\bar{F}(\tau)\sigma_p^2 + \sigma_q^2]^2} \left(N\bar{F}(\tau)\sigma_p^2 \rho \frac{\sigma_q}{\sigma_x} \mu(\tau) - \sigma_q^2 \left[\hat{q} + \rho \frac{\sigma_q}{\sigma_x} (\tau - \bar{x}) \right] \right),$$

where $\mu(\tau) = \frac{\int_{\tau}^{\infty} (x-\tau)f(x) dx}{\bar{F}(\tau)}$. Note that in the last parenthesis, for any $\rho > 0$, $\rho N\bar{F}(\tau)\mu(\tau)$ in the first term is positive and strictly decreasing in τ (because $\mu(\tau)$ is the ‘‘mean residual life’’ of the Normal distribution which is a strictly decreasing quantity) while the second term increases linearly in τ . Therefore, we have $\frac{\partial \bar{q}_u}{\partial \tau} > 0$ for $\tau < \tau^*$, and $\frac{\partial \bar{q}_u}{\partial \tau} < 0$ for $\tau > \tau^*$, where τ^* is the unique solution of

$$\tau = \bar{x} - \frac{\sigma_x \hat{q}}{\sigma_q \rho} + \frac{N\bar{F}(\tau)\sigma_p^2 \mu(\tau)}{\sigma_q^2}. \quad (10)$$

Therefore, $\bar{q}_u(\tau)$ is unimodal with a unique maximum at τ^* ; equivalently, $\bar{q}_u(n_1)$ is unimodal with a unique maximum at $n_1^* = N\bar{F}(\tau^*)$.

As a result, when $N\bar{F}(\tau^*) \in [0, D_1(p)]$ the optimal launch-quantity is $n_1^* = N\bar{F}(\tau^*)$. However, when $N\bar{F}(\tau^*) > D_1(p)$, then by unimodality of \bar{q}_u the optimal inventory quantity decision is $n_1^* = D_1(p)$ (i.e., n_1 as large as possible). Clearly, the latter holds when $p > \tau^*$, while the former holds when $p < \tau^*$.

Proof of Proposition 2

From Proposition 1, we know that if the firm chooses a price which satisfies $p < \tau$, it is optimal to choose availability $n_1 = D_1(\tau) < D_1(p)$. Let \bar{q}_u^* be the best possible SL outcome, i.e., $\bar{q}_u^* = \bar{q}_u(D_1(\tau))$. By contrast, if the firm chooses $p \geq \tau$, then it is optimal to choose availability $n_1 = D_1(p)$.

Fix \bar{q}_u^* to the optimal SL outcome, and consider the following two profit functions: (1) $\pi_1(p) = pN\bar{F}(p - q_u^*)$ and (2) $\pi_2(p) = pN\bar{F}(p - q_u(D_1(p)))$. Thus, (1) assumes an optimal SL outcome irrespective of the price chosen, while (2) assumes that the firm fulfills all demand irrespective of the price chosen. Next, note that from Proposition 1 we have that (i) π_1 describes optimal profit at prices $p \leq \tau^*$ but is infeasible for $p > \tau^*$, and (ii) π_2 describes optimal profit at prices $p \geq \tau^*$ but suboptimal profit for $p < \tau^*$. Thus, the firm's optimal profit at any price p is described by π_1 for $p \in [0, \tau^*]$ and π_2 for $p \in [\tau^*, +\infty)$. It then follows that a necessary and sufficient condition for scarcity to be optimal (in which case the firm chooses $p \in [0, \tau^*]$) is that $\left. \frac{d\pi_1}{dp} \right|_{p=\tau^*} < 0$. To see why, note first that $\pi_1(p) \geq 0$, $\pi_1(0) = 0$ and π_1 is strictly unimodal (this follows from properties of the normal distribution), so that $\left. \frac{d\pi_1}{dp} \right|_{p=\tau^*} < 0$ implies that maximum π_1 (denote π_1^*) is achieved somewhere in $[0, \tau^*]$; second, since $\pi_1 \geq \pi_2 \forall p$ (because $\pi_1 = pN\bar{F}(p - q_u^*) \geq pN\bar{F}(p - q_u(D_1(p))) = \pi_2$) we have $\pi_1^* > \max_{p \in [\tau^*, +\infty)} \pi_2$. Finally, note that $\left. \frac{d\pi_1}{dp} \right|_{p=\tau^*} < 0$ is equivalent to the condition stated in the main result, $\tau^*h(\tau^* - \bar{q}_u^*) > 1$.

Proof of Corollary 1

From Proposition 1, we know that if the firm chooses a price which satisfies $p < \tau$, it is optimal to choose availability $n_1 = D_1(\tau) < D_1(p)$. By contrast, if the firm chooses $p \geq \tau$, then it is optimal to choose availability $n_1 = D_1(p)$. To prove the result, we will compare the best possible policy with price $p < \tau$ against the best possible policy with price $p \geq \tau$.

Consider first policies with $p < \tau$. Any such policy is accompanied by availability $n_1^* = D_1(\tau)$; let $\bar{q}_u^* = \bar{q}_u(n_1^*)$. The firm's pricing problem (within the class of policies with $p < \tau$) is $\max_{p < \tau} Np\bar{F}(p - \bar{q}_u^*)$. The unique solution to this problem is ξ as stated in the main text. Next consider policies with $p > \tau$. Here, any policy is accompanied by availability $n_1 = D_1(p)$. The firm's pricing problem (within the class of policies with $p \geq \tau$) is $\max_{p \geq \tau} Np\bar{F}(p - \bar{q}_u(D_1(p)))$.

Now, note that by unimodality of \bar{q}_u in the proof of Proposition 1, we have that $\max_{p \geq \tau} Np\bar{F}(p - \bar{q}_u(D_1(p))) < \max_{p \geq \tau} Np\bar{F}(p - \bar{q}_u^*)$, because $\bar{q}_u(n_1) < \bar{q}_u^*$ for any $n_1 < D_1(\tau)$ and for $p \geq \tau$ we have that n_1 can be at most $D_1(p)$, which is less than $D_1(\tau)$. Therefore, a necessary and sufficient condition for policy $\{\xi, n_1^*\}$ to be globally optimal is that $\xi < \tau$, because $\xi = \arg \max_p Np\bar{F}(p - \bar{q}_u^*)$. Conversely, if $\xi \geq \tau$, then we have $p^* \in [\tau, +\infty)$ and p^* is found through the one-dimensional problem $\max_p Np\bar{F}(p - \bar{q}_u(D_1(p)))$.

We next show that $\xi < \tau$ holds true provided $\hat{q} < Q$, for some threshold Q which is a function of our model parameters. First note that $\frac{\partial \bar{q}_u}{\partial \hat{q}} > 0$, and therefore $\frac{\partial \bar{q}_u^*}{\partial \hat{q}} > 0$. It follows that the normal hazard ratio $h(p - \bar{q}_u^*)$ is strictly decreasing in \hat{q} , which in turn implies that ξ is strictly increasing in \hat{q} . Next, taking the total derivative of (10) with respect to \hat{q} we have

$$\frac{\partial \tau}{\partial \hat{q}} + \frac{\sigma_x}{\sigma_q \rho} - \frac{\partial}{\partial \tau} \left(\frac{N\bar{F}(\tau)\sigma_p^2\mu(\tau)}{\sigma_q^2} \right) \frac{\partial \tau}{\partial \hat{q}} = 0$$

Note that $\bar{F}(\tau) > 0$, $\frac{\partial \bar{F}}{\partial \tau} < 0$ and $\mu(\tau) > 0$, $\frac{\partial \mu}{\partial \tau} < 0$ ($\mu(\tau)$ is the ‘‘mean residual life’’ function and is decreasing in τ for the normal distribution). Therefore, we have $\frac{\partial \tau}{\partial \hat{q}} < 0$. Since $\frac{\partial \xi}{\partial \hat{q}} > 0$ and $\frac{\partial \tau}{\partial \hat{q}} < 0$, it follows that there exists Q such that $\xi < \tau$ if $\hat{q} < Q$.

Proof of Proposition 3

We establish first the properties of the optimal quantity decision $n_1^* = N\bar{F}(\tau)$, by considering the properties of the threshold τ , which is given by the implicit equation $\tau - \bar{x} + \frac{\sigma_x \hat{q}}{\sigma_q \rho} - \frac{N\bar{F}(\tau)\sigma_p^2 \mu(\tau)}{\sigma_q^2} = 0$. Define $M = \frac{\sigma_x \hat{q}}{\sigma_q \rho}$ and $K(\tau) = \frac{N\bar{F}(\tau)\sigma_p^2 \mu(\tau)}{\sigma_q^2}$ such that τ satisfies

$$\tau - \bar{x} + M - K(\tau) = 0,$$

and note that $K(\tau) > 0$ and $K'(\tau) < 0$ (see proof of Corollary 1). Taking the total derivative with respect to \hat{q} , we have

$$\frac{\partial \tau}{\partial \hat{q}} + \frac{\partial M}{\partial \hat{q}} - \frac{\partial K}{\partial \tau} \frac{\partial \tau}{\partial \hat{q}} = 0.$$

Since $\frac{\partial M}{\partial \hat{q}} > 0$, we have $\frac{\partial \tau}{\partial \hat{q}} < 0$ and it follows that $\frac{\partial n_1^*}{\partial \hat{q}} > 0$. Next, consider the total derivative with respect to ρ .

$$\frac{\partial \tau}{\partial \rho} + \frac{\partial M}{\partial \rho} - \frac{\partial K}{\partial \tau} \frac{\partial \tau}{\partial \rho} = 0.$$

Here, $\frac{\partial M}{\partial \rho} < 0$ (> 0) for $\hat{q} > 0$ (< 0) and therefore $\frac{\partial n_1^*}{\partial \rho} < 0$ (> 0) for $\hat{q} > 0$ (< 0). Next, consider the total derivative with respect to σ_p^2 .

$$\frac{\partial \tau}{\partial \sigma_p^2} - \left(\frac{\partial K}{\partial \sigma_p^2} + \frac{\partial K}{\partial \tau} \frac{\partial \tau}{\partial \sigma_p^2} \right) = 0.$$

Therefore, $\frac{\partial n_1^*}{\partial \sigma_p^2} < 0$.

We now consider the properties of the optimal price p^* . Recall that this is implicitly defined via

$$ph(p - \bar{q}_u^*) - 1 = 0,$$

where $\bar{q}_u^* = \bar{q}_u(n_1^*)$ and it is straightforward that $\frac{\partial \bar{q}_u^*}{\partial \hat{q}} > 0$, $\frac{\partial \bar{q}_u^*}{\partial \rho} > 0$ and $\frac{\partial \bar{q}_u^*}{\partial \sigma_p^2} > 0$. Consider the total derivative with respect to ρ .

$$\left(h(p - \bar{q}_u^*) + p \frac{\partial h(p - \bar{q}_u^*)}{\partial p} \right) \frac{\partial p}{\partial \rho} + \left(p \frac{\partial h(p - \bar{q}_u^*)}{\partial \bar{q}_u^*} \right) \frac{\partial \bar{q}_u^*}{\partial \rho} = 0.$$

Since $h(\cdot)$ is the hazard ratio of a Normal distribution, we have $\frac{\partial h(p - \bar{q}_u^*)}{\partial p} > 0$ and $\frac{\partial h(p - \bar{q}_u^*)}{\partial \bar{q}_u^*} < 0$. Therefore, $\frac{\partial p}{\partial \rho} > 0$. In a similar manner it can be shown that $\frac{\partial p}{\partial \hat{q}} > 0$ and $\frac{\partial p}{\partial \sigma_p^2} > 0$.

Finally, note that the firm’s profit can be expressed as $\pi = Np^* \bar{F}(p^* - q_u^*)$. The properties of the firm’s optimal profit follow readily from the properties of q_u^* as stated above.

Proof of Proposition 4

We will show the desired result by contradiction. Assume an optimal policy $\zeta^* = \{p_1^*, n_1^*, p_2^*\}$ with $n_1^* < D_1(p_1^*)$. The profit under this policy is

$$\pi(p_1^*, n_1^*, p_2^*) = p_1^* n_1^* + \pi_2(n_1^*, p_2^*),$$

where $p_1^* n_1^*$ is the first period profit while $\pi_2(n_1^*, p_2^*)$ is the second period profit. Next, consider policy ψ , $\psi = \{\hat{p}_1, D_1(\hat{p}_1), p_2^*\}$ (i.e., dynamic pricing with unrestricted availability), where \hat{p}_1 is such that $D_1(\hat{p}_1) = n_1^*$. Note that the latter implies $\hat{p}_1 > p_1^*$. Profit under this policy is

$$\begin{aligned}\pi(\hat{p}_1, D_1(\hat{p}_1), p_2^*) &= \pi(\hat{p}_1, n_1^*, p_2^*) \\ &= \hat{p}_1 n_1^* + \pi_2(n_1^*, p_2^*) \\ &> p_1^* n_1^* + \pi_2(n_1^*, p_2^*)\end{aligned}$$

where the last inequality holds because first-period profit under policy ψ is strictly greater than under ζ^* , owing to a price-premium attached to first-period sales. Therefore ζ^* cannot be optimal.

Proof of Proposition 5 Let $\pi_d(p_1, p_2)$ denote profits under dynamic pricing at $\{p_1, p_2\}$ and let $\pi_s(p, n_1)$ denote profits under fixed pricing at $\{p, n_1\}$. We have $\pi_d^* = N p_1^* \int_{p_1^*}^{+\infty} f(x) dx + N p_2^* \int_{\min\{p_1^*, p_2^* - \bar{q}_u(D_1(p_1^*))\}}^{p_1^*} f(x) dx$. Assume that the firm sets $p = p_2^*$ and $n_1 = D_1(p_1^*)$ in a fixed-pricing policy. Then it achieves profit $\pi_s = N p_2^* \int_{p_1^*}^{+\infty} f(x) dx + N p_2^* \int_{\min\{p_1^*, p_2^* - \bar{q}_u(D_1(p_1^*))\}}^{p_1^*} f(x) dx$. Therefore, $\pi_s(p_2^*, D_1(p_1^*)) = \pi_d^* - (p_1^* - p_2^*) D_1(p_1^*)$. Since $\pi_s^* \geq \pi_s(p_2^*, D_1(p_1^*))$, it follows that $\Delta\pi^* = \pi_d^* - \pi_s^* \leq (p_1^* - p_2^*) D_1(p_1^*)$, which is the upper bound of the Proposition.

Next, note that under the optimal fixed-pricing policy $\{p^*, n_1^*\}$ we have $\pi_s^* = p^* n_1^* + N p_2^* \int_{\min\{\tau, p^* - \bar{q}_u(n_1^*)\}}^{\tau} f(x) dx$. Under dynamic pricing, suppose that the firm sets $p_1 = \tau$ such that $D_1(p_1) = n_1^*$ and $p_2 = p^*$. Then it achieves profit $\pi_d(\tau, p^*) = \tau n_1^* + N p_2^* \int_{\min\{\tau, p^* - \bar{q}_u(n_1^*)\}}^{\tau} f(x) dx$. Therefore $\pi_d(\tau, p^*) = \pi_s^* + (\tau - p^*) n_1^*$. Finally, since $\pi_d^* \geq \pi_d(\tau, p^*)$, it follows that $\Delta\pi^* = \pi_d^* - \pi_s^* \geq (\tau - p^*) n_1^*$ which is the lower bound of the Proposition.

Proof of Proposition 7 Let \bar{q}_u denote the posterior mean belief, conditional on n_1 reviews generated in the first period with an average rating of \bar{R} . Under proportional rationing, we have $\bar{q}_u = \frac{n_1 \sigma_p^2}{n_1 \sigma_p^2 + \sigma_q^2} \bar{R} = \frac{n_1 \sigma_p^2}{n_1 \sigma_p^2 + \sigma_q^2} (\hat{q} + \sigma_q \sigma_x h(p))$, that is, \bar{R} is independent of n_1 . If $\hat{q} + \sigma_q \sigma_x h(p) > 0$ then \bar{q}_u is positive and increasing in n_1 , and the firm's problem is $\max_{n_1 \leq D_1(p)} n_1 p + (N \bar{F}(p - \bar{q}_u) - n_1) p = \max_{n_1 \leq D_1(p)} N \bar{F}(p - \bar{q}_u) p$ (note that in this case $D_2 = N \bar{F}(p - \bar{q}_u) - n_1 \geq 0$). The firm's profit is therefore increasing in \bar{q}_u , which itself is increasing in n_1 ; the optimal launch quantity is $n_1^* = D_1(p)$. Next, if $\hat{q} + \sigma_q \sigma_x h(p) < 0$ then \bar{q}_u is negative and decreasing in n_1 . Therefore, the number of overall sales are bounded from above at $N \bar{F}(p)$; these sales are achieved by selling to all customers willing to purchase in the first period; this implies $n_1^* = D_1(p)$. Note that a profit-equivalent policy is to sell no units in the first period, and sell $D_1(p)$ units in the second period; any other n_1 such that $n_1 \in (0, D_1(p))$ is suboptimal. \square

C. Numerical Investigation of Profit Function Approximation

For the case of efficient rationing, we compare the solution and optimal profit obtained analytically for problem (7) with the corresponding values obtained numerically for problem (6), for the 420 scenarios used in Table 1: $N \in \{1000, 3000, 5000, 7000\}$; $q_p = 0$, $\sigma_p = 0.1$, $\hat{q} \in \{-0.15, -0.1, -0.05, 0, 0.05, 0.1, 0.15\}$; $\sigma_x \in \{0.4, 0.6, 0.8\}$, $\sigma_q = 0.6$, $\bar{x} = 0.5$; $\rho \in \{0.1, 0.15, 0.2, 0.25, 0.3\}$. We report summary statistics of this comparison in the following table (the maximum errors were observed under scenarios of low product quality, i.e., for $\hat{q} = -0.15$).

	average error	max error	% of scenarios with < 1% error
Optimal profit π^*	0.01%	0.63%	100%
Optimal price p^*	0.16%	3.14%	97.4%
Optimal segmentation threshold τ^* (note $n_1^* = D_1(\tau^*)$)	0.11%	4.8%	96.7%

D. Bayesian Social Learning

We describe here the procedure by which individual product reviews can be de-biased by rational agents when information on reviewer preferences is readily available. Consider an individual buyer whose idiosyncratic component is x_i . According to (1), the buyer's review is generated from a Normal source $N(\hat{q} + \rho \frac{\sigma_q}{\sigma_x} (x_i - \bar{x}), \sigma_q^2(1 - \rho^2))$. Since our objective is to learn \hat{q} , we must first subtract the deterministic bias term $\rho \frac{\sigma_q}{\sigma_x} (x_i - \bar{x})$ from the buyer's review, thus "centering" the review around \hat{q} . Next, employing the standard Gaussian updating model (Chamley 2004), the Bayesian posterior mean (after observing a single review generated by a customer with idiosyncratic preference component x_i) is given by

$$q_u = \frac{\sigma_p^2}{\sigma_p^2 + \sigma_q^2(1 - \rho^2)} \left(q_i - \rho \frac{\sigma_q}{\sigma_x} (x_i - \bar{x}) \right), \quad (11)$$

where q_i is the buyer's review. If a set S of reviews is available with $|S| = n_s$, the same procedure can be repeated, updating q_u sequentially after de-biasing each review. The end result is

$$q_u = \frac{n_s \sigma_p^2}{n_s \sigma_p^2 + \sigma_q^2(1 - \rho^2)} \left(\frac{1}{n_s} \sum_{i \in S} q_i - \rho \frac{\sigma_q}{\sigma_x} \left(\frac{1}{n_s} \sum_{i \in S} x_i - \bar{x} \right) \right) \quad (12)$$

$$= \frac{n_s \sigma_p^2}{n_s \sigma_p^2 + \sigma_q^2(1 - \rho^2)} \left(R_s - \rho \frac{\sigma_q}{\sigma_x} (\bar{x}_s - \bar{x}) \right), \quad (13)$$

with $R_s = \frac{1}{n_s} \sum_{i \in S} q_i$ and $\bar{x}_s = \frac{1}{n_s} \sum_{i \in S} x_i$.

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